

A Particular Class of Penti-Hexagonal Polyhedra

GERALD CHAYT AND HERBERT HAUPTMAN

*Applied Mathematics Branch
Mathematics and Information Sciences Division*

August 22, 1968

(Reprinted with corrections)



NAVAL RESEARCH LABORATORY
Washington, D.C.



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ABSTRACT

In accordance with the Navy's interest in developing an underwater spherical vessel capable of human occupancy, studies have been made of a particular class of convex polyhedra as a possible approximation to a sphere. The class referred to is a category of penti-hexagonal polyhedra, formed from the regular dodecahedron by inserting successive layers of equilateral convex hexagons within the basic pentagonal structure in a manner which maximizes the congruence and the symmetry of the polyhedral surfaces.

The structure of a particular polyhedron in this class is determined by computing the vertex angles of all the distinct hexagons composing its surface, i.e., the polyhedron face angles. With this end in mind, trigonometric equations are derived for all hexagons having less than a certain degree of symmetry, and all dihedral angles whose edges are not perpendicular to a plane of symmetry of the polyhedron. These equations are then solved using a Newton-Raphson-type process. The structure of all polyhedra in this class up to 3242 faces has been determined, and the number of trigonometric equations in each case exactly equals the number of unknown angles. It is suspected that this equality is maintained for higher order polyhedra in this class.

After the relevant trigonometric equations and the techniques for solving them have been derived, it is seen that, in practically all instances, the value of the solution, if convergence occurs, is independent of the initial guess used for the face angles.

Trends among certain face angles become evident for progressively higher order PH polyhedra, which permit calculating approximate diameters as a multiple of the edge length; for polyhedra having up to 3242 faces, the values of all face angles, hexagon diagonals, and dihedral angles may also be determined.

Two conjectures emerge from the study, and these take the form of two theorems which are based on six preliminary definitions.

PROBLEM STATUS

This is a final report on one phase of the problem; work on other phases continues.

AUTHORIZATION

NRL Problem B01-03
Project RR 003-05-41-5060

Manuscript submitted February 17, 1968.

A PARTICULAR CLASS OF PENTI-HEXAGONAL POLYHEDRA

INTRODUCTION

It is currently of interest to the Navy to construct a large sphere capable of both human occupancy and deep-sea submergence. The required size of this sphere necessitates that it be built from many small components which, from a machinist's viewpoint, should be as congruent and symmetrical as possible.

This report describes a preliminary phase of this problem, that of approximating the sphere by a particular class of convex polyhedra. The numerical parameters of these polyhedra are given, and the methods of calculating them are described. The polyhedra in this class have a progressively increasing number of faces and, as this number increases, the polyhedra more closely resemble a sphere, i.e., the dihedral angles formed at all edges approach 180° and the sum of the face angles at each vertex approaches 360° . This phase is relevant to the general problem, because, given a sphere whose center coincides with the centroid of the polyhedron, the vertices of the polyhedron can be radially projected onto the sphere and can be connected by arcs of great circles. The spherical polygons thus formed can be thought of as the required components. However, the question of the congruence and symmetry of these components has yet to be resolved.

THEORETICAL DISCUSSION

Description of Polyhedra

As is standard, we refer to the plane polygons which compose a polyhedron as *faces* of the polyhedron; the intersection of two faces is called an *edge*, and the intersection of three or more edges is called a *vertex* of the polyhedron.

There are a number of properties that we specify for these polyhedra. First, we require that they be convex. We also require that they be as symmetrical as possible in structure, which implies maximum possible congruence and symmetry among the vertices, edges, and faces. Finally, we wish to have exactly three faces meet at each vertex. This last property is desired because a vertex having this structure has its associated dihedral angles uniquely determined by its face angles; if four or more faces meet at a vertex, there is some degree of freedom in forming the associated dihedral angles, leading to a certain amount of instability in the structure (analogous to the fact that three sides uniquely determine a triangle, but four or more sides do not uniquely determine the associated polygon).

We will be concerned here with a class of penti-hexagonal polyhedra, formed from the regular dodecahedron by inserting successive layers of equilateral convex hexagons within the basic pentagonal structure. These hexagons are constructed and inserted in a manner consistent with the aforementioned properties. A member of this class will be referred to as a *PH polyhedron*.

Both the questions of the existence and the solutions of these polyhedra interest us. Surprisingly, the latter question is more easily answered, since the digital computer can

be effectively used. To affirm the existence of a particular polyhedron, however, we must actually construct it.

In a regular dodecahedron we say that two pentagons are adjacent if they meet in an edge; we extend this concept to a PH polyhedron and call two pentagons adjacent if they would be adjacent under a "collapse" of the structure to a dodecahedron, i.e., if all hexagons were withdrawn from the polyhedron and the resulting pentagons were drawn together in an order-preserving manner to form a regular dodecahedron. For a given PH polyhedron let n be the number of hexagons inserted between adjacent pentagons. Then the number of faces N of the polyhedron is $N = 12 + 10n(n+2)$.

Figures 1 and 2 show PH polyhedra with 42 and 92 faces ($n = 1$ and 2, respectively). These are the only PH polyhedra we have constructed to date.

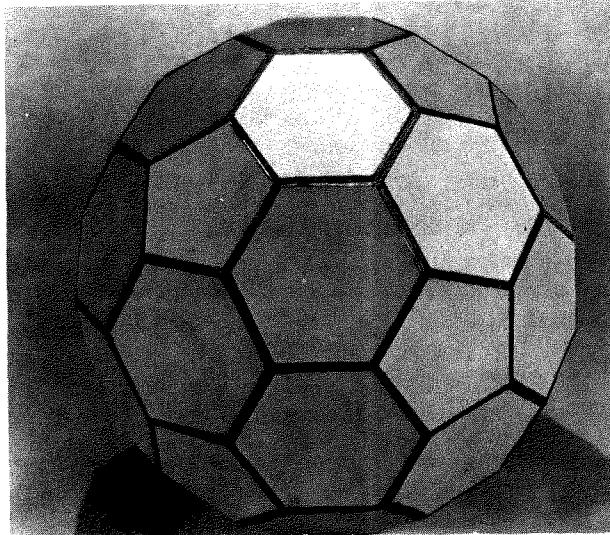


Fig. 1 - A PH polyhedron having 42 faces

Equations for a PH Polyhedron

A polyhedron is solved by computing all vertex angles of all the hexagons composing the faces. This is theoretically possible if the number of unknown angles is not less than the number of equations obtainable from the polyhedron. Our experience thus far shows that for all PH polyhedra up to 3242 faces ($n = 1, 2, \dots, 17$), the number of available equations equals the number of unknown angles. This leads us to suspect that all PH polyhedra have a solution and, furthermore, that the number of equations always equals the number of unknown angles. The equations for a typical PH polyhedron are of two basic types: (a) dihedral-angle equations and (b) equilateral hexagon equations.

Dihedral-Angle Equations — A dihedral-angle equation results from the fact that each dihedral angle can usually be expressed in two ways, in terms of the face angles at each of the two vertices bounding the edge which forms the dihedral angle. An exception occurs when the two vertices are symmetric, i.e., when the face angles at one vertex are respectively equal to those at the other. This occurs when the edge forming the dihedral angle is perpendicular to and is bisected by a plane of symmetry of the polyhedron. We

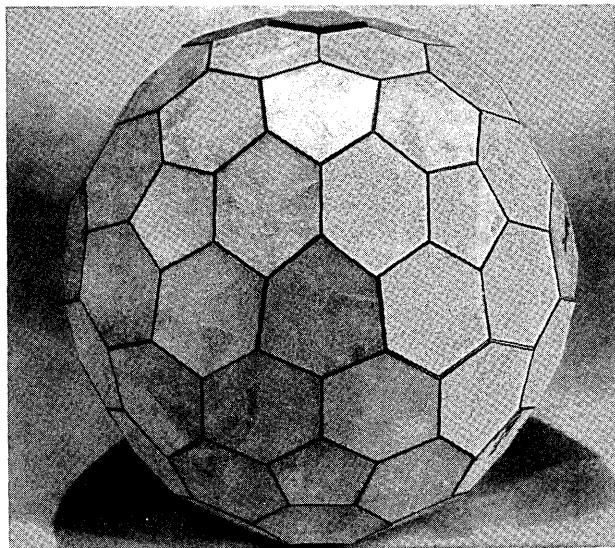


Fig. 2 - A PH polyhedron having 92 faces

thus define a *solvable* dihedral angle as one for which the associated vertices are not symmetric. When we equate the two expressions for a solvable dihedral angle, the value of the dihedral angle is eliminated, and we have an expression relating the six associated face angles.

Figure 3a indicates a dihedral angle formed at the intersection of faces B and C. The edge of intersection terminates in a vertex at which faces A, B, and C meet, and also in a vertex at which faces B, C, and D meet. Consider the upper vertex. We let α_1 , α_2 , and α_3 denote the positive or negative excesses over 120° ($2\pi/3$ radians) of the face angles of faces A, B, and C, respectively, at this vertex. That is, the face angles of A, B, and C at the upper vertex are respectively equal to $(2\pi/3) + \alpha_1$, $(2\pi/3) + \alpha_2$, and $(2\pi/3) + \alpha_3$ radians. We define α_4 , α_5 , and α_6 similarly at the lower vertex. If we

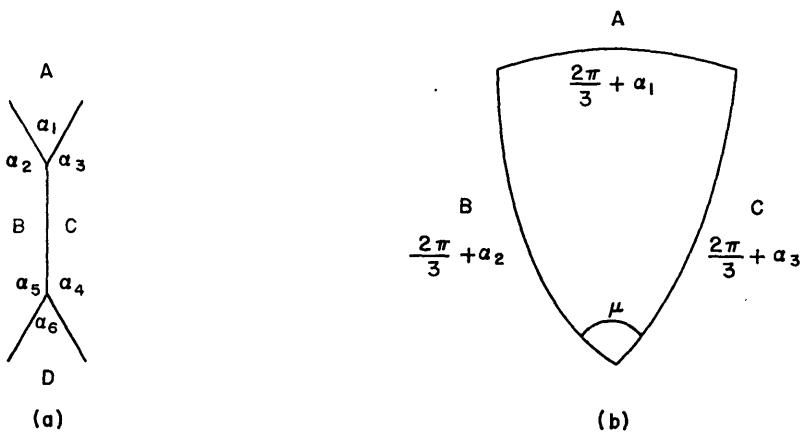


Fig. 3 - The dihedral angle formed by two intersecting faces of a polyhedron: (a) actual view and (b) spherical representation

consider the upper vertex as the center of a sphere, the intersections of faces A, B, and C with the surface of the sphere are arcs of great circles and form a spherical triangle as shown in Fig. 3b. The respective angles of arc of this triangle are thus $(2\pi/3) + \alpha_1$, $(2\pi/3) + \alpha_2$, and $(2\pi/3) + \alpha_3$. We see that μ is the dihedral angle of interest, and, from the cosine law of spherical trigonometry,

$$\cos \left(\frac{2\pi}{3} + \alpha_1 \right) = \cos \left(\frac{2\pi}{3} + \alpha_2 \right) \cos \left(\frac{2\pi}{3} + \alpha_3 \right) + \sin \left(\frac{2\pi}{3} + \alpha_2 \right) \sin \left(\frac{2\pi}{3} + \alpha_3 \right) \cos \mu,$$

so that

$$\cos \mu = \frac{\cos \left(\frac{2\pi}{3} + \alpha_1 \right) - \cos \left(\frac{2\pi}{3} + \alpha_2 \right) \cos \left(\frac{2\pi}{3} + \alpha_3 \right)}{\sin \left(\frac{2\pi}{3} + \alpha_2 \right) \sin \left(\frac{2\pi}{3} + \alpha_3 \right)}.$$

Similarly, employing the spherical triangle BCD,

$$\cos \mu = \frac{\cos \left(\frac{2\pi}{3} + \alpha_6 \right) - \cos \left(\frac{2\pi}{3} + \alpha_4 \right) \cos \left(\frac{2\pi}{3} + \alpha_5 \right)}{\sin \left(\frac{2\pi}{3} + \alpha_4 \right) \sin \left(\frac{2\pi}{3} + \alpha_5 \right)}.$$

Eliminating $\cos \mu$ and clearing of fractions, we have

$$\begin{aligned} & \left[\cos \left(\frac{2\pi}{3} + \alpha_1 \right) - \cos \left(\frac{2\pi}{3} + \alpha_2 \right) \cos \left(\frac{2\pi}{3} + \alpha_3 \right) \right] \sin \left(\frac{2\pi}{3} + \alpha_4 \right) \sin \left(\frac{2\pi}{3} + \alpha_5 \right) \\ & - \left[\cos \left(\frac{2\pi}{3} + \alpha_6 \right) - \cos \left(\frac{2\pi}{3} + \alpha_4 \right) \cos \left(\frac{2\pi}{3} + \alpha_5 \right) \right] \sin \left(\frac{2\pi}{3} + \alpha_2 \right) \sin \left(\frac{2\pi}{3} + \alpha_3 \right) = 0. \quad (1) \end{aligned}$$

This is the general dihedral-angle equation, and is clearly not an identity for a solvable angle.

Equilateral-Hexagon Equations — Unless stated otherwise, all hexagons are assumed equilateral and convex. For the general equilateral convex hexagon with no symmetry, three angles are sufficient to determine it completely. Since our convention will be to label all six angles differently, it is possible to obtain three independent relations among these angles. Two sets of such relations are derived here.

The first set of relations is completely trigonometric. To derive them, consider the hexagon in Fig. 4 having sides of unit length. Let the vertex angles be labeled ϕ_1 , ϕ_2 , ϕ_3 , ψ_1 , ψ_2 , and ψ_3 ; let S_1 , S_2 , and S_3 be the lengths of the sides of the triangle formed by connecting alternate vertices, as shown, and let β_1 , β_2 , and β_3 be the opposite angles. Then, by the sine law for plane triangles,

$$\frac{S_1}{\sin \beta_1} = \frac{S_2}{\sin \beta_2} = \frac{S_3}{\sin \beta_3}. \quad (2)$$

After inverting and changing the sign of Eq. (2), and substituting

$$S_i = 2 \sin \frac{\phi_i}{2}, \quad i = 1, 2, 3,$$

we have

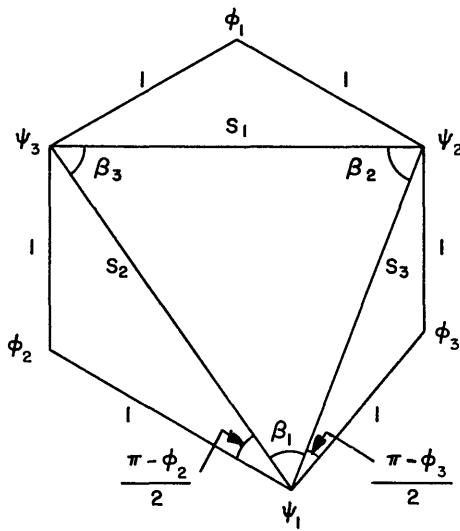


Fig. 4 - Hexagon having sides of unit length

$$-\frac{\sin \beta_1}{\sin \frac{\phi_1}{2}} = -\frac{\sin \beta_2}{\sin \frac{\phi_2}{2}} = -\frac{\sin \beta_3}{\sin \frac{\phi_3}{2}} \quad (3)$$

From Fig. 4 it follows that

$$\beta_1 = \psi_1 - \left(\frac{\pi - \phi_2}{2} + \frac{\pi - \phi_3}{2} \right) = \psi_1 + \frac{\phi_2 + \phi_3}{2} - \pi$$

Hence,

$$-\sin \beta_1 = \sin (\beta_1 + \pi) = \sin \left(\psi_1 + \frac{\phi_2 + \phi_3}{2} \right)$$

We can similarly show that

$$-\sin \beta_2 = \sin \left(\psi_2 + \frac{\phi_1 + \phi_3}{2} \right)$$

and

$$-\sin \beta_3 = \sin \left(\psi_3 + \frac{\phi_1 + \phi_2}{2} \right)$$

Substituting these expressions into Eq. (3) we get

$$\frac{\sin \left(\psi_1 + \frac{\phi_2 + \phi_3}{2} \right)}{\sin \frac{\phi_1}{2}} = \frac{\sin \left(\psi_2 + \frac{\phi_1 + \phi_3}{2} \right)}{\sin \frac{\phi_2}{2}} = \frac{\sin \left(\psi_3 + \frac{\phi_1 + \phi_2}{2} \right)}{\sin \frac{\phi_3}{2}} \quad (4)$$

It follows in exactly the same manner that

$$\frac{\sin \left(\phi_1 + \frac{\psi_2 + \psi_3}{2} \right)}{\sin \frac{\psi_1}{2}} = \frac{\sin \left(\phi_2 + \frac{\psi_1 + \psi_3}{2} \right)}{\sin \frac{\psi_2}{2}} = \frac{\sin \left(\phi_3 + \frac{\psi_1 + \psi_2}{2} \right)}{\sin \frac{\psi_3}{2}} \quad (5)$$

Equations (4) and (5) yield the three desired independent relations.

The second set of relations is partially trigonometric, the trigonometric portion following as a by-product of computing a long diagonal of an equilateral hexagon. We will change our labeling system slightly and denote the angle at vertex i by β_i (Fig. 5a). As in the case of the dihedral-angle equation, we define α_i to be the excess of β_i over $2\pi/3$ radians, i.e., $\beta_i = \alpha_i + (2\pi/3)$, $i = 1, 2, \dots, 6$. Let L_{ij} denote the length of the diagonal connecting vertices i and j . We shall now compute L_{13} and L_{14} (Fig. 5b). We see immediately that

$$L_{13} = 2 \sin \frac{\beta_2}{2}$$

and

$$\begin{aligned} L_{14} &= 1 + L_{13}^2 - 2L_{13} \cos \gamma = 1 + L_{13}^2 - 2L_{13} \cos \left(\beta_3 + \frac{\beta_2}{2} - \frac{\pi}{2} \right) \\ &= 1 + L_{13}^2 - 2L_{13} \sin \left(\beta_3 + \frac{\beta_2}{2} \right) = 1 + 4 \sin^2 \frac{\beta_2}{2} - 4 \sin \frac{\beta_2}{2} \sin \left(\beta_3 + \frac{\beta_2}{2} \right) \end{aligned} \quad (6)$$

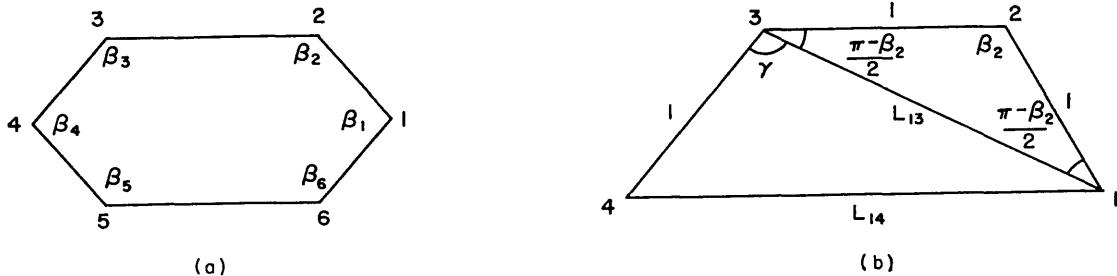


Fig. 5 - Diagram used in computing the long diagonal of an equilateral hexagon: (a) equilateral hexagon and (b) enlarged view of the upper half of the hexagon

Substituting into Eq. (6) the trigonometric identities

$$\sin \frac{\beta_2}{2} \sin \left(\beta_3 + \frac{\beta_2}{2} \right) = \frac{1}{2} [\cos \beta_3 - \cos (\beta_2 + \beta_3)]$$

and

$$4 \sin^2 \frac{\beta_2}{2} = 2 - 2 \cos \beta_2,$$

we have

$$L_{14} = 3 - 2 [\cos \beta_2 + \cos \beta_3 - \cos (\beta_2 + \beta_3)]. \quad (7)$$

Since we can compute L_{14} equally well using β_5 and β_6 , we also have

$$L_{14} = 3 - 2 [\cos \beta_5 + \cos \beta_6 - \cos (\beta_5 + \beta_6)]. \quad (8)$$

It follows from Eqs. (7) and (8) that

$$\cos \beta_2 + \cos \beta_3 - \cos (\beta_2 + \beta_3) = \cos \beta_5 + \cos \beta_6 - \cos (\beta_5 + \beta_6),$$

or, equivalently,

$$\begin{aligned} & \cos\left(\frac{2\pi}{3} + \alpha_2\right) + \cos\left(\frac{2\pi}{3} + \alpha_3\right) - \cos\left(\frac{4\pi}{3} + \alpha_2 + \alpha_3\right) \\ &= \cos\left(\frac{2\pi}{3} + \alpha_5\right) + \cos\left(\frac{2\pi}{3} + \alpha_6\right) - \cos\left(\frac{4\pi}{3} + \alpha_5 + \alpha_6\right) \end{aligned} \quad (9)$$

This will be referred to as the four-angle equation. It has been derived with reference to the long diagonal connecting vertices 1 and 4. A hexagon with no symmetry has three independent equations of this type, one equation for each long diagonal, and these could comprise our set. We also have the option of using only two of these equations and supplementing them with the relations

$$\begin{aligned} \beta_1 + \beta_2 + \beta_3 + \beta_4 + \beta_5 + \beta_6 &= 4\pi, \\ \text{or} \\ \alpha_1 + \alpha_2 + \alpha_3 + \alpha_4 + \alpha_5 + \alpha_6 &= 0, \end{aligned} \quad (10)$$

which will be called the sum-to-zero equation. This latter option is chosen because Eq. (10) is a linear relation, and this option is also more compatible with the case of a hexagon with onefold symmetry, as described below.

Symmetric Hexagons — A convex equilateral hexagon with onefold symmetry, i.e., symmetry about an axis passing through one and only one pair of opposite vertices, is completely determined by two of its angles. Figure 6 shows such a hexagon, labeled with both the conventions of Figs. 4 and 5a. Since we have only four different angles, we need two relations among them. We see that Eqs. (4) and (5) become

$$\frac{\sin(\psi_1 + \phi_2)}{\sin \frac{\phi_1}{2}} = \frac{\sin\left(\psi_2 + \frac{\phi_1 + \phi_2}{2}\right)}{\sin \frac{\phi_2}{2}} \quad (4')$$

and

$$\frac{\sin(\phi_1 + \psi_2)}{\sin \frac{\psi_1}{2}} = \frac{\sin\left(\phi_2 + \frac{\psi_1 + \psi_2}{2}\right)}{\sin \frac{\psi_2}{2}}, \quad (5')$$

yielding the two required relations. On the other hand, using the four-angle equations is complicated by the existence of only one such equation, since the equations generated about L_{25} and L_{36} are the same and that generated about L_{14} is an identity. Hence, this equation

$$\begin{aligned} & \cos\left(\frac{2\pi}{3} + \alpha_1\right) + \cos\left(\frac{2\pi}{3} + \alpha_2\right) - \cos\left(\frac{4\pi}{3} + \alpha_1 + \alpha_2\right) \\ &= \cos\left(\frac{2\pi}{3} + \alpha_3\right) + \cos\left(\frac{2\pi}{3} + \alpha_4\right) - \cos\left(\frac{4\pi}{3} + \alpha_3 + \alpha_4\right) \end{aligned} \quad (9')$$

must be supplemented with the sum-to-zero equation, i.e.,

$$\alpha_1 + 2(\alpha_2 + \alpha_3) + \alpha_4 = 0. \quad (10')$$

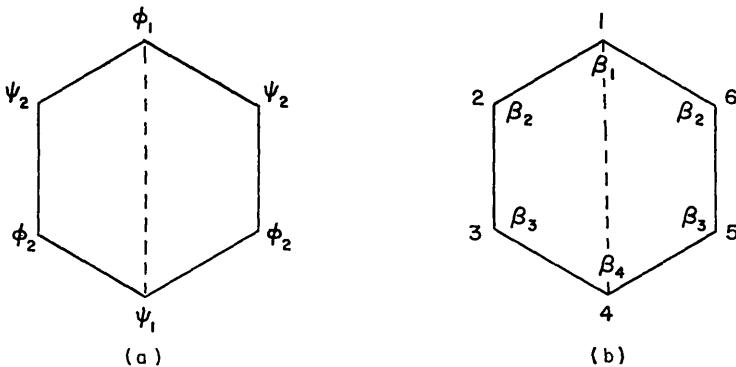


Fig. 6 - Convex hexagon with onefold symmetry:
(a) labeled according to the convention used in
Fig. 4 and (b) labeled according to the convention
used in Fig. 5a

Figures 7a and 7b show hexagons with twofold and threefold symmetry, respectively. Both these types of hexagons are completely determined by one angle. Since, in each case, only one basic symbol is used (ϵ or ω , the excesses of the respective vertex angles over $2\pi/3$ radians), no other relations are required (as with the case of the α angles).

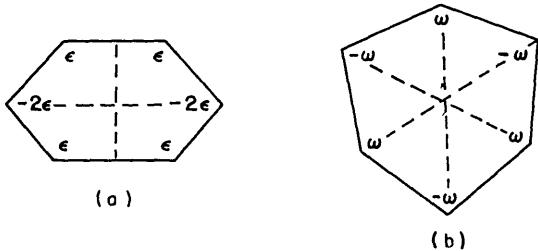


Fig. 7 - Hexagons having twofold and
threefold symmetry: (a) hexagon with
twofold symmetry and (b) hexagon with
threefold symmetry

COMPUTATIONAL TECHNIQUES

For any of the PH polyhedra with no more than 3242 faces, the combined equations for all distinct solvable dihedral angles and for all distinct types of equilateral hexagons form a system of equations, mostly nonlinear, whose number exactly equals the number of unknown face angles. The solution of these equations must be performed on a digital computer; the program to do this must use some approximation technique and should require only a minimum of input data. The technique we describe was devised with these goals in mind. The details of the computer program content and use are described in Appendix A.

Solution of Polyhedron Equations

Suppose our system of equations involves m unknowns $\alpha_1, \dots, \alpha_m$, each of which is the excess of some unknown angle β_i over $2\pi/3$ radians, i.e., $\beta_i = \alpha_i + (2\pi/3)$, $i = 1, \dots, m$. If we have some initial guess of the values of these unknowns, say $\alpha'_1, \dots, \alpha'_m$, then these can possibly be used in the system of equations to yield increments $\delta\alpha_i$, $i = 1, \dots, m$ such that $\alpha_i = \alpha'_i + \delta\alpha_i$, $i = 1, \dots, m$ is a better approximation to the true solution. We could then consider $\alpha_1, \dots, \alpha_m$ as a new initial guess and repeat the process indefinitely until some criteria for convergence is satisfied. Experience has shown that this Newton-Raphson-type method works for a "correct" initial guess. For most of the PH polyhedra

we solved, double-precision computation was used (about 25 significant figures), but for the highest order polyhedra (>2252 faces) the limitations of core storage space in the computer forced us to perform some of the computations in single-precision (about 11 significant figures).

To use the above method, the system of equations must be converted into a set of approximate linear equations of the form

$$AX = B, \quad (11)$$

where A is an $m \times m$ coefficient matrix, X is an $m \times 1$ matrix of unknown increments, i.e.,

$$X^T = (\delta\alpha_1 \ \delta\alpha_2 \ \dots \ \delta\alpha_m) \quad (12)$$

(the T superscript means transpose), and B is an $m \times 1$ matrix. Once the system, Eq. (11), is obtained, the solution X may be found by one of several methods available for linear equations. Such a conversion is possible, and we refer to it as a linearization process. We shall show briefly how this process was used on the particular system of equations we chose: the dihedral-angle equations combined with the four-angle equations and the sum-to-zero equations. The basic idea of this process rests on the assumption that α'_i nearly equals α_i , the true solution, so that $\delta\alpha_i = \alpha_i - \alpha'_i$ is a relatively small number. Then we can make the approximation

$$\begin{aligned} \sin\left(\frac{2\pi}{3} + \alpha_i\right) &= \sin\left(\frac{2\pi}{3} + \alpha'_i + \delta\alpha_i\right) \\ &= \sin\left(\frac{2\pi}{3} + \alpha'_i\right) \cos \delta\alpha_i + \cos\left(\frac{2\pi}{3} + \alpha'_i\right) \sin \delta\alpha_i \\ &\approx \sin\left(\frac{2\pi}{3} + \alpha'_i\right) + \cos\left(\frac{2\pi}{3} + \alpha'_i\right) \delta\alpha_i, \end{aligned} \quad (13)$$

since

$$\cos \delta\alpha_i \approx 1 \quad \text{and} \quad \sin \delta\alpha_i \approx \delta\alpha_i.$$

For notational simplicity we define

$$S_{\alpha_i} \equiv \sin\left(\frac{2\pi}{3} + \alpha'_i\right), \quad C_{\alpha_i} \equiv \cos\left(\frac{2\pi}{3} + \alpha'_i\right).$$

Thus, Eq. (13) becomes

$$\sin\left(\frac{2\pi}{3} + \alpha_i\right) \approx S_{\alpha_i} + C_{\alpha_i} \delta\alpha_i. \quad (14)$$

Similarly, we have

$$\cos\left(\frac{2\pi}{3} + \alpha_i\right) \approx C_{\alpha_i} - S_{\alpha_i} \delta\alpha_i. \quad (15)$$

Substituting Eqs. (14) and (15) into Eq. (1) we have

$$\begin{aligned} & \left[C_{\alpha_1} - S_{\alpha_1} \delta\alpha_1 - \left(C_{\alpha_2} - S_{\alpha_2} \delta\alpha_2 \right) \left(C_{\alpha_3} - S_{\alpha_3} \delta\alpha_3 \right) \right] \left(S_{\alpha_4} + C_{\alpha_4} \delta\alpha_4 \right) \left(S_{\alpha_5} + C_{\alpha_5} \delta\alpha_5 \right) \\ & - \left[C_{\alpha_6} - S_{\alpha_6} \delta\alpha_6 - \left(C_{\alpha_4} - S_{\alpha_4} \delta\alpha_4 \right) \left(C_{\alpha_5} - S_{\alpha_5} \delta\alpha_5 \right) \right] \left(S_{\alpha_2} + C_{\alpha_2} \delta\alpha_2 \right) \left(S_{\alpha_3} + C_{\alpha_3} \delta\alpha_3 \right) = 0, \end{aligned}$$

which becomes, after collecting like terms and discarding terms of higher than first-order approximation,

$$\begin{aligned} & - S_{\alpha_1} S_{\alpha_4} S_{\alpha_5} \delta\alpha_1 + \left[S_{\alpha_2} C_{\alpha_3} S_{\alpha_4} S_{\alpha_5} - \left(C_{\alpha_6} - C_{\alpha_4} C_{\alpha_5} \right) C_{\alpha_2} S_{\alpha_3} \right] \delta\alpha_2 \\ & + \left[C_{\alpha_2} S_{\alpha_3} S_{\alpha_4} S_{\alpha_5} - \left(C_{\alpha_6} - C_{\alpha_4} C_{\alpha_5} \right) S_{\alpha_2} C_{\alpha_3} \right] \delta\alpha_3 + \left[\left(C_{\alpha_1} - C_{\alpha_2} C_{\alpha_3} \right) C_{\alpha_4} S_{\alpha_5} - S_{\alpha_2} S_{\alpha_3} S_{\alpha_4} C_{\alpha_5} \right] \delta\alpha_4 \\ & + \left[\left(C_{\alpha_1} - C_{\alpha_2} C_{\alpha_3} \right) S_{\alpha_4} C_{\alpha_5} - S_{\alpha_2} S_{\alpha_3} C_{\alpha_4} S_{\alpha_5} \right] \delta\alpha_5 + S_{\alpha_2} S_{\alpha_3} S_{\alpha_6} \delta\alpha_6 \\ & = - \left(C_{\alpha_1} - C_{\alpha_2} C_{\alpha_3} \right) S_{\alpha_4} S_{\alpha_5} + \left(C_{\alpha_6} - C_{\alpha_4} C_{\alpha_5} \right) S_{\alpha_2} S_{\alpha_3}. \end{aligned} \quad (16)$$

To avoid confusion, observe that the unknowns $\delta\alpha_1, \dots, \delta\alpha_6$ in Eq. (16) are actually the increments of the six associated face angles of the dihedral angle shown in Fig. 3a but are not necessarily the first six elements of the sequence composing X^T in Eq. (12). Indeed, to be strictly accurate, the angles in Eq. (16) should be of the form

$$\alpha_{i_1}, \delta\alpha_{i_1}, \alpha_{i_2}, \delta\alpha_{i_2}, \dots, \alpha_{i_6}, \text{ and } \delta\alpha_{i_6}, \text{ where } 1 \leq i_j \leq m, j = 1, \dots, 6. \quad (17)$$

However, we will retain the informal notation of Eq. (16) and also use it in subsequent approximate equations, with the expectation that no confusion will arise.

Equation (16) is the linearized, dihedral-angle equation. Using the same informal notation, the linearization of the four-angle equation, Eq. (9), for L_{14} and the rearrangement of the sum-to-zero equation, Eq. (10), results in the relations

$$\begin{aligned} & - \left(S_{\alpha_2} - S_{\alpha_2} C_{\alpha_3} - C_{\alpha_2} S_{\alpha_3} \right) \delta\alpha_2 - \left(S_{\alpha_3} - S_{\alpha_2} C_{\alpha_3} - C_{\alpha_2} S_{\alpha_3} \right) \delta\alpha_3 + \left(S_{\alpha_5} - S_{\alpha_5} C_{\alpha_6} - C_{\alpha_5} S_{\alpha_6} \right) \delta\alpha_5 \\ & + \left(S_{\alpha_6} - S_{\alpha_5} C_{\alpha_6} - C_{\alpha_5} S_{\alpha_6} \right) \delta\alpha_6 = - \left(C_{\alpha_2} + C_{\alpha_3} - C_{\alpha_2} C_{\alpha_3} + S_{\alpha_2} S_{\alpha_3} \right) + \left(C_{\alpha_5} + C_{\alpha_6} - C_{\alpha_5} C_{\alpha_6} + S_{\alpha_5} S_{\alpha_6} \right) \end{aligned} \quad (18)$$

and

$$\delta\alpha_1 + \delta\alpha_2 + \delta\alpha_3 + \delta\alpha_4 + \delta\alpha_5 + \delta\alpha_6 = - \alpha'_1 - \alpha'_2 - \alpha'_3 - \alpha'_4 - \alpha'_5 - \alpha'_6. \quad (19)$$

The system of equations defined by Eq. (11) is formed, in part, by applying Eq. (16) to all distinct solvable dihedral angles. Another part of the system results from applying Eq. (19) to all distinct hexagons with either onefold symmetry or no symmetry. The remainder of the system is generated by applying Eq. (18) and a similar equation for another long diagonal to each distinct hexagon with no symmetry, and by applying the linearization of Eq. (9') to each distinct hexagon with onefold symmetry.

We now return to the problem of choosing a correct initial guess for the solution $\alpha_1, \dots, \alpha_m$ of the polyhedron equations. There are two methods to be subsequently described: (a) the guess based on the solution of a set of approximate linear equations, and (b) the constant-value initial guess.

Approximate Linear Equations — The method of approximate linear equations is based on the assumptions that (a) the sum of the face angles at any vertex is almost the same as that at any other vertex, and (b) all hexagons are almost regular. Although these assumptions are sometimes fallacious, they serve to provide good starting values of the angles (still expressed as excesses over $2\pi/3$ radians).

To implement assumption (a), we equate the sums of the face angles at all the distinct vertices (say V of them) of the polyhedron, giving us $V - 1$ linear equations. This will be illustrated shortly.

Using assumption (b), we may derive three independent linear equations for the nonsymmetric hexagon (these reduce to two equations for hexagons with onefold symmetry). Let an equilateral hexagon be labeled as in Fig. 8 with ϵ_1 , ϵ_2 , ϵ_3 , η_1 , η_2 , and η_3 representing excesses over $2\pi/3$ radians. Assume further that these excesses are near 0 radians, i.e., that the hexagon is nearly regular. Using Eq. (9), with η_3 , ϵ_2 , ϵ_3 , and η_2 , respectively, replacing α_2 , α_3 , α_5 , and α_6 , we have

$$\begin{aligned} & \cos\left(\frac{2\pi}{3} + \eta_3\right) + \cos\left(\frac{2\pi}{3} + \epsilon_2\right) - \cos\left(\frac{4\pi}{3} + \eta_3 + \epsilon_2\right) \\ &= \cos\left(\frac{2\pi}{3} + \epsilon_3\right) + \cos\left(\frac{2\pi}{3} + \eta_2\right) - \cos\left(\frac{4\pi}{3} + \epsilon_3 + \eta_2\right) \end{aligned} \quad (20)$$

But

$$\begin{aligned} \cos\left(\frac{2\pi}{3} + \eta_3\right) &= \cos \frac{2\pi}{3} \cos \eta_3 - \sin \frac{2\pi}{3} \sin \eta_3 \\ &= -\frac{1}{2} \cos \eta_3 - \frac{\sqrt{3}}{2} \sin \eta_3 \\ &\approx -\frac{1}{2} - \frac{\sqrt{3}}{2} \eta_3, \end{aligned}$$

since $\cos \eta_3 \approx 1$ and $\sin \eta_3 \approx \eta_3$. Similarly,

$$\begin{aligned} \cos\left(\frac{2\pi}{3} + \epsilon_2\right) &\approx -\frac{1}{2} - \frac{\sqrt{3}}{2} \epsilon_2, \\ \cos\left(\frac{2\pi}{3} + \epsilon_3\right) &\approx -\frac{1}{2} - \frac{\sqrt{3}}{2} \epsilon_3, \end{aligned}$$

and

$$\cos\left(\frac{2\pi}{3} + \eta_2\right) \approx -\frac{1}{2} - \frac{\sqrt{3}}{2} \eta_2.$$

We also have

$$\cos\left(\frac{4\pi}{3} + \eta_3 + \epsilon_2\right) = \cos \frac{4\pi}{3} \cos (\eta_3 + \epsilon_2) - \sin \frac{4\pi}{3} \sin (\eta_3 + \epsilon_2) \approx -\frac{1}{2} + \frac{\sqrt{3}}{2} (\eta_3 + \epsilon_2)$$

and

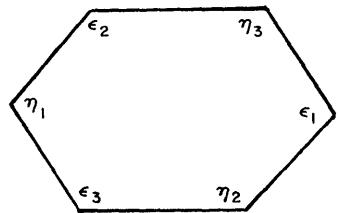


Fig. 8 - Nonsym-metric hexagon

$$\cos \left(\frac{4\pi}{3} + \epsilon_3 + \eta_2 \right) \approx -\frac{1}{2} + \frac{\sqrt{3}}{2} (\epsilon_3 + \eta_2).$$

Substituting these approximations back into Eq. (20) yields

$$\frac{1}{2} \left[-1 - 2\sqrt{3} (\eta_3 + \epsilon_2) \right] \approx \frac{1}{2} \left[-1 - 2\sqrt{3} (\epsilon_3 + \eta_2) \right],$$

or

$$\eta_3 + \epsilon_2 \approx \epsilon_3 + \eta_2. \quad (21)$$

Similarly, we can replace α_2 , α_3 , α_5 , and α_6 in Eq. (9) with ϵ_2 , η_1 , η_2 , and ϵ_1 , respectively, and get

$$\epsilon_2 + \eta_1 \approx \eta_2 + \epsilon_1. \quad (22)$$

Combining Eqs. (21) and (22) with the sum-to-zero equation

$$\epsilon_1 + \epsilon_2 + \epsilon_3 + \eta_1 + \eta_2 + \eta_3 = 0$$

yields

$$\epsilon_1 \approx \frac{1}{3} \eta_1 - \frac{2}{3} (\eta_2 + \eta_3), \quad (23a)$$

$$\epsilon_2 \approx \frac{1}{3} \eta_2 - \frac{2}{3} (\eta_3 + \eta_1), \quad (23b)$$

and

$$\epsilon_3 \approx \frac{1}{3} \eta_3 - \frac{2}{3} (\eta_1 + \eta_2). \quad (23c)$$

These are the three desired independent, approximate relations, expressing ϵ_1 , ϵ_2 , and ϵ_3 as functions of η_1 , η_2 , and η_3 .

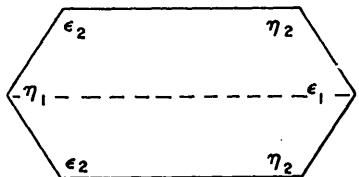


Fig. 9 - Hexagon with onefold symmetry

For a hexagon with onefold symmetry (Fig. 9) we express the relations differently. Equations (23) become

$$\epsilon_1 \approx \frac{1}{3} \eta_1 - \frac{4}{3} \eta_2$$

and

$$\epsilon_2 \approx -\frac{1}{3} \eta_2 - \frac{2}{3} \eta_1.$$

or

$$\epsilon_2 \approx \frac{\epsilon_1 - 3\eta_1}{4} \quad (24a)$$

and

$$\eta_2 \approx \frac{\eta_1 - 3\epsilon_1}{4}, \quad (24b)$$

thereby expressing ϵ_2 and η_2 as functions of ϵ_1 and η_1 .

If we use Eqs. (23) for all nonsymmetric hexagons, Eqs. (24) for all hexagons with onefold symmetry, and equate the sum of the face angles at each distinct vertex of a PH polyhedron with no more than 252 faces, we get a system of m equations in m unknowns which yields a good initial approximation for $\alpha_1, \dots, \alpha_m$. If the PH polyhedron has 362 faces, we get one less equation than the number of unknowns. Although this can be circumvented by guessing at a value for the common sum of the face angles, care must still be taken to avoid inconsistency in the equations. An acceptable solution can be obtained by guessing at the value of one of the hexagon vertex angles. Anticipating similar difficulties for PH polyhedra of higher order, we stopped using this method for obtaining initial values. Instead, a much simpler technique was adopted, which will now be described.

The Constant-Value Initial Guess — Experimentation revealed that convergence can be obtained by setting most of the angles (excesses) equal to a correctly chosen constant initial value. A few such initial values were tested, and Table 1 indicates the results obtained. As can be seen, convergence can be obtained for all PH polyhedra investigated by setting all initial excesses equal to any of several constant values in the range from 1×10^{-6} to 1.0 degree. Furthermore, for a given polyhedron, a unique solution is obtained which is independent of the initial guess used in this range.* However, Table 1 also indicates that, as the order of the polyhedra increases, convergence seems to fail at progressively lower initial values. We will now show some sample calculations.

Polyhedron with 42 Faces — Figure 10a shows the basic structure of the surface for the polyhedron having 42 faces, and Fig. 10b illustrates the only distinct solvable dihedral angle. All symbols denote excesses over $2\pi/3$ radians; thus, the pentagon vertex angle has an excess of $108^\circ - 120^\circ = -12^\circ$ ($-\pi/15$ radians). Since all hexagons are congruent (labeled with 1 in Fig. 10a) and have twofold symmetry, no equilateral-hexagon equations exist or are necessary. Thus, the only equation for this polyhedron is that for the dihedral angle shown in Fig. 10b, which is

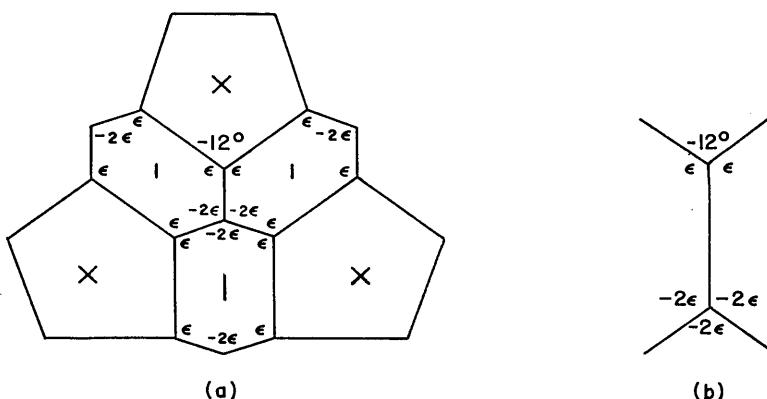


Fig. 10 - Surface structure for a polyhedron having 42 faces: (a) surface portion between three mutually adjacent pentagons and (b) solvable dihedral angle

*An exception occurs for the 42-faced PH polyhedron, for which an initial guess of 20° leads to a nonconvex solution.

Table 1
Penti-Hexagonal Polyhedra Convergence of Iterations to
Solution for All Angles Equal to a Constant Initial Value

Initial Angle Excess (degrees)	Convergence* for Various Numbers of Faces												2892†	3242†		
	42	92	162	252	362	492	642	812	1002	1212	1442	1692	1962	2252		
0	C	C	C	C	I	I	I	I	I	I	I	I	I	C	C	C
1×10^{-6}	C	C	C	C	C	C	C	C	C	C	C	C	C	C	C	C
1×10^{-5}	C	C	C	C	C	C	C	C	C	C	C	C	C	C	C	C
1×10^{-3}	C	C	C	C	C	C	C	C	C	C	C	C	C	C	C	C
1×10^{-2}	C	C	C	C	C	C	C	C	C	C	C	C	C	C	C	C
1×10^{-1}	C	C	C	C	C	C	C	C	C	C	C	C	C	C	C	C
1	C	C	C	C	C	C	C	C	C	C	C	C	C	C	C	F
10	C	C	C	C	C	C	C	F	F	F	F	F	F	F	F	F
20	C_N	F	C	C	I	C	C	F	F	C	F	F	F	F	I	F
40	F	F	F	F	F	F	F	F	F	F	F	F	F	I	F	F

*Key: C = converges to solution; I = inconsistent or dependent set of equations; F = equations are consistent and independent but convergence is not obtained in 30 iterations; C_N = convergence, but to a nonconvex solution.
†Single precision.

$$\begin{aligned} & \left[\cos \frac{3\pi}{5} - \cos^2 \left(\frac{2\pi}{3} + \epsilon \right) \right] \sin^2 \left(\frac{2\pi}{3} - 2\epsilon \right) - \cos \left(\frac{2\pi}{3} - 2\epsilon \right) \\ & \times \left[1 - \cos \left(\frac{2\pi}{3} - 2\epsilon \right) \right] \sin^2 \left(\frac{2\pi}{3} + \epsilon \right) = 0. \end{aligned} \quad (25)$$

From Eqs. (14) and (15) we have

$$\sin \left(\frac{2\pi}{3} + \epsilon \right) \approx S_\epsilon + C_\epsilon \delta\epsilon \quad (26a)$$

and

$$\cos \left(\frac{2\pi}{3} + \epsilon \right) \approx C_\epsilon - S_\epsilon \delta\epsilon, \quad (26b)$$

and a similar approximation yields

$$\sin \left(\frac{2\pi}{3} - 2\epsilon \right) \approx S_{2\epsilon} - 2C_{2\epsilon} \delta\epsilon \quad (27a)$$

and

$$\cos \left(\frac{2\pi}{3} - 2\epsilon \right) \approx C_{2\epsilon} + 2S_{2\epsilon} \delta\epsilon, \quad (27b)$$

where

$$S_{2\epsilon} \equiv \sin \left(\frac{2\pi}{3} - 2\epsilon' \right), \quad (28a)$$

$$C_{2\epsilon} \equiv \cos \left(\frac{2\pi}{3} - 2\epsilon' \right), \quad (28b)$$

$$\epsilon = \epsilon' + \delta\epsilon. \quad (29)$$

Substituting these approximations into Eq. (25) yields the linearized equation

$$2 \left[2S_{2\epsilon}C_{2\epsilon} \left(1 - \cos \frac{3\pi}{5} \right) + S_\epsilon(C_\epsilon - S_\epsilon S_{2\epsilon} - C_\epsilon C_{2\epsilon}) \right] \delta\epsilon = - \left(\cos \frac{3\pi}{5} - C_\epsilon^2 \right) S_{2\epsilon}^2 + C_{2\epsilon}(1 - C_{2\epsilon}) S_\epsilon^2 \quad (30)$$

We will use this equation with an initial guess obtained by the approximate linear equations. Thus, we will equate the sums of the face angles at each of the two distinct vertices and get

$$-12^\circ + 2\epsilon = -6\epsilon,$$

or

$$\epsilon = 1.5^\circ.$$

Setting $\epsilon' = \epsilon$, Eq. (30) becomes

$$-4.709064053 \delta\epsilon = -0.0178252483,$$

which yields, after conversion from radians to degrees,

$$\delta\epsilon = 0.21688^\circ$$

and

$$\epsilon = \epsilon' + \delta\epsilon = 1.71688^\circ.$$

Again, setting $\epsilon' = \epsilon$, Eq. (30) becomes

$$-4.683353111 \delta\epsilon = -0.0000485857,$$

which yields

$$\epsilon = \epsilon' + \delta\epsilon = 1.7174744^\circ.$$

Repeating the process once more, we see that

$$|\delta\epsilon| < 10^{-8} \text{ degrees.}$$

Thus, the last computed value of ϵ is correct to seven decimal places.

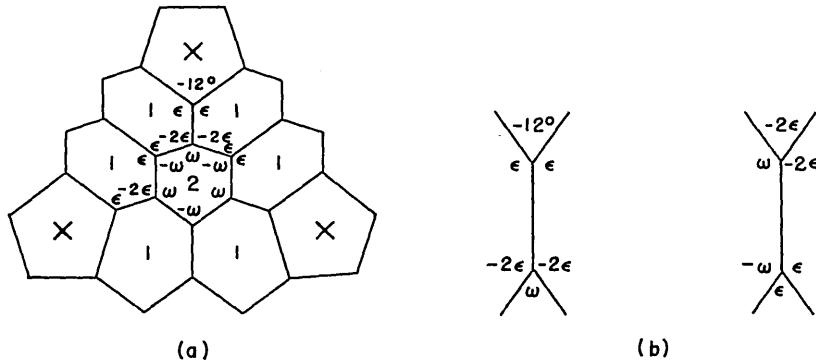


Fig. 11 - Surface structure for a polygon having 92 faces: (a) surface portion between three mutually adjacent pentagons and (b) solvable dihedral angles

Polyhedron with 92 Faces — Figure 11a shows the basic structure of the surface for the polyhedron having 92 faces, and Fig. 11b illustrates the only two distinct solvable dihedral angles. As in the previous example, all symbols denote excesses over $2\pi/3$ radians. For this polyhedron there are two types of hexagons, labeled 1 and 2, which have twofold and threefold symmetry, respectively. Hence, it is true again that only the dihedral-angle equations are available and necessary for the solution of this polyhedron. These equations are

$$\left[\cos \frac{3\pi}{5} - \cos^2 \left(\frac{2\pi}{3} + \epsilon \right) \right] \sin^2 \left(\frac{2\pi}{3} - 2\epsilon \right) - \left[\cos \left(\frac{2\pi}{3} + \omega \right) - \cos^2 \left(\frac{2\pi}{3} - 2\epsilon \right) \right] \sin^2 \left(\frac{2\pi}{3} + \epsilon \right) = 0 \quad (31a)$$

and

$$\begin{aligned} & \cos \left(\frac{2\pi}{3} - 2\epsilon \right) \left[1 - \cos \left(\frac{2\pi}{3} + \omega \right) \right] \sin \left(\frac{2\pi}{3} + \epsilon \right) \sin \left(\frac{2\pi}{3} - \omega \right) \\ & - \cos \left(\frac{2\pi}{3} + \epsilon \right) \left[1 - \cos \left(\frac{2\pi}{3} - \omega \right) \right] \sin \left(\frac{2\pi}{3} - 2\epsilon \right) \sin \left(\frac{2\pi}{3} + \omega \right) = 0. \end{aligned} \quad (31b)$$

Using Eqs. (26), (27), (28), (29), and

$$\sin \left(\frac{2\pi}{3} + \omega \right) \approx S_\omega + C_\omega \delta\omega, \quad (32a)$$

$$\cos\left(\frac{2\pi}{3} + \omega\right) \approx C_\omega - S_\omega \delta\omega, \quad (32b)$$

$$\sin\left(\frac{2\pi}{3} - \omega\right) \approx S_{-\omega} - C_{-\omega} \delta\omega, \quad (33a)$$

and

$$\cos\left(\frac{2\pi}{3} - \omega\right) \approx C_{-\omega} + S_{-\omega} \delta\omega, \quad (33b)$$

where

$$S_\omega \equiv \sin\left(\frac{2\pi}{3} + \omega'\right), \quad (34a)$$

$$C_\omega \equiv \cos\left(\frac{2\pi}{3} + \omega'\right), \quad (34b)$$

$$S_{-\omega} \equiv \sin\left(\frac{2\pi}{3} - \omega'\right), \quad (34c)$$

$$C_{-\omega} \equiv \cos\left(\frac{2\pi}{3} - \omega'\right), \quad (34d)$$

and

$$\omega = \omega' + \delta\omega, \quad (35)$$

Eqs. (31) become, respectively,

$$2 \left[S_\epsilon C_\epsilon (1 - C_\omega) + 2 S_{2\epsilon} C_{2\epsilon} \left(1 - \cos \frac{3\pi}{5} \right) \right] \delta\epsilon + S_\omega S_\epsilon^2 \delta\omega = (C_\omega - C_{2\epsilon}^2) S_\epsilon^2 - \left(\cos \frac{3\pi}{5} - C_\epsilon^2 \right) S_{2\epsilon}^2 \quad (36a)$$

and

$$\begin{aligned} & [S_{-\omega} (1 - C_\omega) (2 S_\epsilon S_{2\epsilon} + C_\epsilon C_{2\epsilon}) + S_\omega (1 - C_{-\omega}) (S_\epsilon S_{2\epsilon} + 2 C_\epsilon C_{2\epsilon})] \delta\epsilon \\ & - [S_\epsilon S_\omega S_{-\omega} + S_\epsilon C_{2\epsilon} C_{-\omega} (1 - C_\omega) + C_\epsilon S_{2\epsilon} C_\omega (1 - C_{-\omega})] \delta\omega \\ & = -C_{2\epsilon} (1 - C_\omega) S_\epsilon S_{-\omega} + C_\epsilon (1 - C_{-\omega}) S_{2\epsilon} S_\omega. \end{aligned} \quad (36b)$$

As before, we will use an initial guess obtained by the approximate linear equations. Since none of the hexagons have less than twofold symmetry, there are no equations having the form of Eq. (24) available, as was also the case in the previous example. Thus, it is sufficient to equate the sums of the face angles at each distinct vertex and obtain

$$-12^\circ + 2\epsilon = -4\epsilon + \omega = 2\epsilon - \omega,$$

from which

$$\epsilon = 4^\circ \quad \text{and} \quad \omega = 12^\circ$$

Setting $\epsilon' = \epsilon$ and $\omega' = \omega$, Eqs. (36) become

$$-3.366230173 \delta\epsilon + 0.5107658885 \delta\omega = -0.02187645948$$

and

$$3.928272966 \delta\epsilon - 1.200259980 \delta\omega = -0.01136725444,$$

from which

$$\delta\epsilon = 0.9032296584^\circ$$

and

$$\delta\omega = 3.498765621^\circ,$$

after conversion from radians to degrees. Thus,

$$\epsilon = \epsilon' + \delta\epsilon = 4.903229658^\circ \quad \text{and} \quad \omega = \omega' + \delta\omega = 15.49876562^\circ$$

Again, setting $\epsilon' = \epsilon$ and $\omega' = \omega$, Eqs. (36) become

$$-3.304276457 \delta\epsilon + 0.4714392410 \delta\omega = 0.0007218427490$$

and

$$3.901795987 \delta\epsilon - 1.156908223 \delta\omega = -0.001100754205,$$

which yield, after conversion from radians to degrees,

$$\epsilon = \epsilon' + \delta\epsilon = 4.894095802^\circ$$

and

$$\omega = \omega' + \delta\omega = 15.52247547^\circ.$$

Repeating this process yields

$$\epsilon = 4.894099098^\circ$$

and

$$\omega = 15.52248782^\circ,$$

and again repeating the process reveals that

$$|\delta\epsilon| < 10^{-8} \text{ degrees} \quad \text{and} \quad |\delta\omega| < 10^{-8} \text{ degrees};$$

therefore, the above results are correct to seven decimal places.

The computations for the 42-faced polyhedron were performed on a LOCI-2 desk computer, and those for the 92-faced polyhedron were performed on a CDC 3800 digital computer.

Automatic Methods of Solution

As can be inferred from the examples just given, the number of equations for a PH polyhedron increases rapidly as the number of faces increases, and linearization of these equations would soon become excessively tedious if one were to attempt this for some of the more advanced PH polyhedra. Fortunately, however, there are better methods for the solution of these polyhedra, using digital computers. The first method to be described is that of *automatic linearization and use of auxiliary angles*. This will be followed by a description of the *chain method*, an improvement upon the first method.

Automatic Linearization and Use of Auxiliary Angles — For brevity, we will subsequently use the term "angle" to denote the excess of an actual angle over $2\pi/3$ radians, unless specifically stated otherwise. It is apparent that the linearized equations, Eqs. (30), (36a), and (36b) have different forms, and we could rewrite them as the functional equations

$$g_{30}(\epsilon, \delta\epsilon) = b_{30}(\epsilon),$$

$$g_{36a}(\epsilon, \omega, \delta\epsilon, \delta\omega) = b_{36a}(\epsilon, \omega),$$

and

$$g_{36b}(\epsilon, \omega, \delta\epsilon, \delta\omega) = b_{36b}(\epsilon, \omega).$$

It would appear that each new dihedral angle encountered in the quest for solutions of higher order polyhedra may well be represented by a trigonometric equation of a form different from those encountered previously, thereby necessitating repeatedly the tedious linearization process. If we define, using Eq. (16), the functions

$$\begin{aligned} g(\alpha_1, \dots, \alpha_6, \delta\alpha_1, \dots, \delta\alpha_6) &\equiv -S_{\alpha_1} S_{\alpha_4} S_{\alpha_5} \delta\alpha_1 + \left[S_{\alpha_2} C_{\alpha_3} S_{\alpha_4} S_{\alpha_5} - \left(C_{\alpha_6} - C_{\alpha_4} C_{\alpha_5} \right) C_{\alpha_2} S_{\alpha_3} \right] \delta\alpha_2 \\ &+ \left[C_{\alpha_2} S_{\alpha_3} S_{\alpha_4} S_{\alpha_5} - \left(C_{\alpha_6} - C_{\alpha_4} C_{\alpha_5} \right) S_{\alpha_2} C_{\alpha_3} \right] \delta\alpha_3 \\ &+ \left[\left(C_{\alpha_1} - C_{\alpha_2} C_{\alpha_3} \right) C_{\alpha_4} S_{\alpha_5} - S_{\alpha_2} S_{\alpha_3} S_{\alpha_4} C_{\alpha_5} \right] \delta\alpha_4 \\ &+ \left[\left(C_{\alpha_1} - C_{\alpha_2} C_{\alpha_3} \right) S_{\alpha_4} C_{\alpha_5} - S_{\alpha_2} S_{\alpha_3} C_{\alpha_4} S_{\alpha_5} \right] \delta\alpha_5 + S_{\alpha_2} S_{\alpha_3} S_{\alpha_6} \delta\alpha_6 \end{aligned} \quad (37a)$$

and

$$b(\alpha_1, \dots, \alpha_6) \equiv -\left(C_{\alpha_1} - C_{\alpha_2} C_{\alpha_3} \right) S_{\alpha_4} S_{\alpha_5} + \left(C_{\alpha_6} - C_{\alpha_4} C_{\alpha_5} \right) S_{\alpha_2} S_{\alpha_3}, \quad (37b)$$

then the functional equation for the general dihedral angle is

$$g(\alpha_1, \dots, \alpha_6, \delta\alpha_1, \dots, \delta\alpha_6) = b(\alpha_1, \dots, \alpha_6), \quad (38)$$

which is linear in $\delta\alpha_1, \dots, \delta\alpha_6$. Furthermore, Eqs. (30), (36a), and (36b) can be expressed as

$$g(-12^\circ, \epsilon, \epsilon, -2\epsilon, -2\epsilon, -2\epsilon, 0, \delta\epsilon, \delta\epsilon, -2\delta\epsilon, -2\delta\epsilon, -2\delta\epsilon) = b(-12^\circ, \epsilon, \epsilon, -2\epsilon, -2\epsilon, -2\epsilon),$$

$$g(-12^\circ, \epsilon, \epsilon, -2\epsilon, -2\epsilon, \omega, 0, \delta\epsilon, \delta\epsilon, -2\delta\epsilon, -2\delta\epsilon, \delta\omega) = b(-12^\circ, \epsilon, \epsilon, -2\epsilon, -2\epsilon, \omega),$$

and

$$g(-2\epsilon, \omega, -2\epsilon, \epsilon, -\omega, \epsilon, -2\delta\epsilon, \delta\omega, -2\delta\epsilon, \delta\epsilon, -\delta\omega, \delta\epsilon) = b(-2\epsilon, \omega, -2\epsilon, \epsilon, -\omega, \epsilon).$$

If we now define

$$\alpha_3 \equiv -2\epsilon, \quad (39a)$$

$$\alpha_4 \equiv -\omega, \quad (39b)$$

and

$$\alpha_5 \equiv -12^\circ, \quad (39c)$$

Eqs. (30), (36a), and (36b) are replaced, respectively, by the systems of equations

$$\begin{aligned} g(\alpha_5, \epsilon, \epsilon, \alpha_3, \alpha_3, \alpha_3, \delta\alpha_5, \delta\epsilon, \delta\epsilon, \delta\alpha_3, \delta\alpha_3, \delta\alpha_3) &= b(\alpha_5, \epsilon, \epsilon, \alpha_3, \alpha_3, \alpha_3) \\ 2\delta\epsilon + \delta\alpha_3 &= 0 \\ \delta\alpha_5 &= 0, \end{aligned}$$

$$g(\alpha_5, \epsilon, \epsilon, \alpha_3, \alpha_3, \omega, \delta\alpha_5, \delta\epsilon, \delta\epsilon, \delta\alpha_3, \delta\alpha_3, \delta\omega) = b(\alpha_5, \epsilon, \epsilon, \alpha_3, \alpha_3, \omega)$$

$$2\delta\epsilon + \delta\alpha_3 = 0$$

$$\delta\alpha_5 = 0,$$

and

$$g(\alpha_3, \omega, \alpha_3, \epsilon, \alpha_4, \epsilon, \delta\alpha_3, \delta\omega, \delta\alpha_3, \delta\epsilon, \delta\alpha_4, \delta\epsilon) = b(\alpha_3, \omega, \alpha_3, \epsilon, \alpha_4, \epsilon)$$

$$2\delta\epsilon + \delta\alpha_3 = 0$$

$$\delta\omega + \delta\alpha_4 = 0$$

$$\delta\alpha_5 = 0.$$

In each case we can deduce the arguments of the g and b functions simply by inspecting the six face angles associated with a given dihedral angle; these arguments then replace the dummy parameters in Eqs. (37), which can then be used to compute the coefficients of $\delta\alpha_1, \dots, \delta\alpha_6$ in the left-hand side of Eq. (38) and the value of the right-hand side of Eq. (38) in each of the three systems by means of a FORTRAN subroutine. Hence, no more analytic linearization is required once Eq. (16) has been derived. Thus, introducing auxiliary variables (e.g., those defined in Eqs. (39)) facilitates the use of a single general linearized equation for all dihedral angles encountered in any PH polyhedron. Each auxiliary variable requires an extra equation to be added to the original system, but this is a small price to pay for the elimination of manual linearization.

In the same way, we can use Eq. (18) and introduce auxiliary angles, if necessary, to provide a single, general, linearized four-angle equation. Since Eq. (19) is already linear, we see that we can standardize the solution of the whole system of approximate linear equations; i.e., we do not have to perform any new analytical work to evaluate all elements of A and B in Eq. (11) for any PH polyhedron of arbitrary complexity.

For illustration, let us consider the PH polyhedron with 1002 faces, in which nine layers of hexagons are inserted between adjacent pentagons. Figure 12 shows the portion of this polyhedron existing between three mutually adjacent pentagons. The same configuration exists over the rest of the polyhedral surface, i.e., the basic pattern shown is congruent to the pattern between any other three mutually adjacent pentagons. The pentagons are denoted by x 's in their centers; the hexagons are denoted by numbers in their centers, those with the same number being congruent. Hexagons 1 through 5 have twofold symmetry (Fig. 7a), those numbered 6, 10, 12, and 13 have onefold symmetry (Fig. 6), and the remainder have no symmetry. Use of the linearized dihedral-angle, four-angle, and sum-to-zero equations results in a system of 51 linear equations in the unknowns $\delta\alpha_1, \dots, \delta\alpha_{51}$, corresponding to the angles $\alpha_1, \dots, \alpha_{51}$ whose subscripts are shown in the face-angle positions in the upper-right-hand portion of Fig. 12.

Observe that the pattern of hexagons within the framework of the three mutually adjacent pentagons actually contains six congruent regions. Also observe that the hexagons numbered 6, 10, and 13 that are vertically aligned have dotted lines drawn through their axes of symmetry. The same is true for the rightmost hexagon numbered 12. These dotted lines, together with the edges that join them, form a boundary of one such region (in this case the boundaries intersect at the common vertex of the type-13 hexagons). The information available within and on the boundaries of this or any of the other five regions is sufficient to generate the entire system of equations, and this is why we need label the face angles in only one region. That is, all possible dihedral angles and hexagon types are contained within a single region. We shall concentrate on the region in which the face angles are labeled, referred to subsequently as the *principal region*.

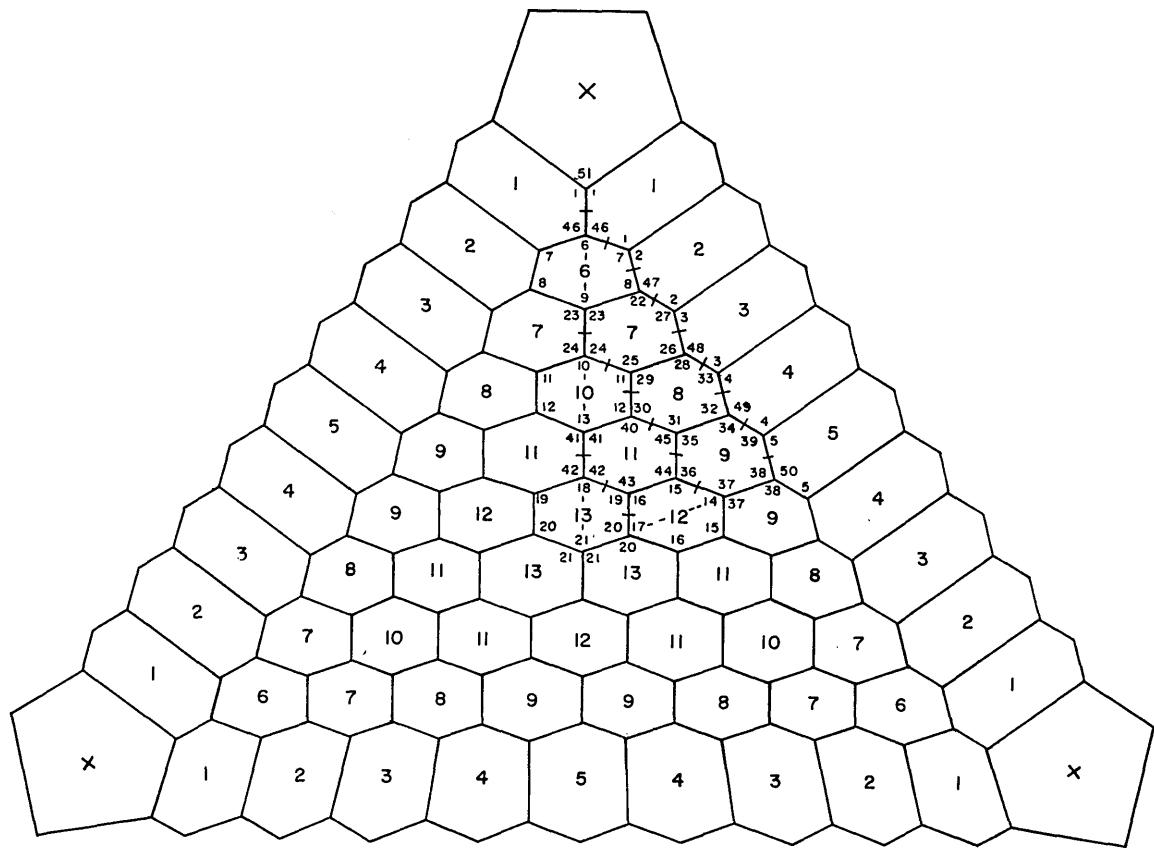


Fig. 12 - PH polyhedron having 1002 faces with nine layers of hexagons between adjacent pentagons

The description of the chain method must be preceded by a discussion of the method of furnishing dihedral-angle information to the computer. Figure 13 is the typical dihedral angle shown in Fig. 3a but with the angles relabeled to conform to Eq. (17). The ordered subscripts of these angles are sufficient information to describe the dihedral angle completely. This information is relayed to the computer by a call to the FORTRAN subroutine DIHD by means of the instruction

```
CALL DIHD (i1, i2, i3, i4, i5, i6),
```

where i_1, \dots, i_6 are either integer variables or constants corresponding to the subscripts in Fig. 13. This subroutine, when called, calculates the coefficients of the unknowns $\delta\alpha_{i_1}, \dots, \delta\alpha_{i_6}$ in the left-hand side of Eq. (38) and also the right-hand side of Eq. (38). Similar subroutines are available for calculating the parameters of the linearized four-angle equations and sum-to-zero equations. For instance, for the 1002-faced polyhedron (Fig. 12) the instruction

```
CALL DIHD (51, 1, 1, 46, 46, 6)
```

would result in a calculation of the parameters of the linearized equation for the dihedral angle involving the pentagon and hexagon types 1 and 6. To calculate the parameters for all 25 distinct solvable dihedral angles in our example, we would have to call DIHD 25 times, deducing the correct set of ordered parameters each time. Although tolerable for this case, the number of distinct solvable dihedral angles increases for higher order

polyhedra and makes the task of generating all the different calls to DIHD more difficult. Thus, the automatic linearization process, although reducing significantly the amount of manual labor required, needs to be augmented by further refinements. The principal refinement is the chain method.

Chain Method — Consider the principal region in Fig. 12. We shall loosely refer to the boundary that passes through the axes of symmetry of hexagon types 6, 10, and 13 as the *vertical boundary*, and that passing through the axis of symmetry of hexagon type 12 as the *diagonal boundary*. We shall say that two edges of the polyhedron that meet at a vertex are *connected*. We define a *chain* as a sequence of connected edges, beginning with an edge on the vertical boundary and terminating at the diagonal boundary in such a manner that no face of the polyhedron is bounded by more than two edges of any one chain. The edges which compose the chain are called *branches*, and the vertices along the chain are called *nodes*; thus, the number of nodes of a chain is always one greater than the number of branches. We also allow in our definition a chain consisting of a single point — one node and no branches. In Fig. 12 the chains are illustrated by placing crossbars in their branches; thus, the 1002-faced polyhedron has four distinct chains, one of which is a chain of one node — at the vertex common to the three type-13 hexagons.

Fig. 13 - Typical dihedral angle used in the chain method

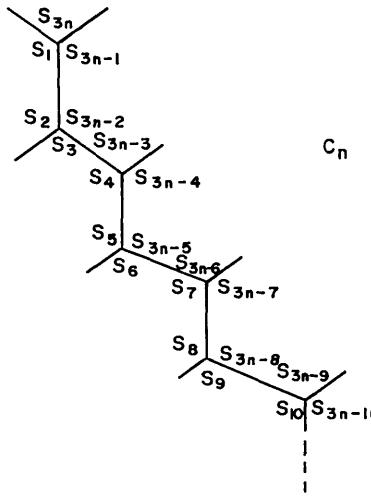
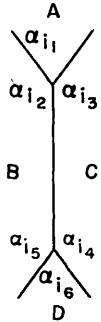


Fig. 14 - Chain having n nodes

A typical chain is shown in Fig. 14. The number of nodes this chain has is n , and we use the symbol C_n to denote such a chain. The symbols S_1, S_2, \dots, S_{3n} represent the subscripts of the face angles encountered while traversing a counterclockwise path around the chain. This path terminates at the topmost face angle, as shown in Fig. 14. Thus, for the topmost (and longest) chain in Fig. 12, the sequence S_1, S_2, \dots, S_{3n} becomes 1, 46, 6, 7, 8, 22, 27, 26, 28, 33, 32, 34, 39, 38, 38, 50, 5, 4, 49, 4, 3, 48, 3, 2, 47, 2, 1, 46, 1, and 51. To generate the parameters of Eq. (38) for the dihedral angle along the topmost branch of a chain, we issue the FORTRAN instruction

```
CALL DIHD (S3n, S1, S3n-1, S3n-2, S2, S3).
```

For the dihedral angle corresponding to the third branch from the top, the required call is

```
CALL DIHD (S3n-3, S4, S3n-4, S3n-5, S5, S6),
```

which leads to an algorithm for generating the subroutine calls for alternating branches starting with the topmost branch. This is

CALL DIHD ($S_{3n-3k+3}$, S_{3k-2} , $S_{3n-3k+2}$, $S_{3n-3k+1}$, S_{3k-1} , S_{3k}),

for $k = 1, 2, \dots$, until $3k$ exceeds $3n - 3k + 1$. A similar algorithm exists for the subroutine calls for alternating branches starting with the second branch from the top:

CALL DIHD (S_{3k-1} , S_{3k} , $S_{3n-3k+1}$, S_{3n-3k} , S_{3k+1} , $S_{3n-3k-1}$),

for $k = 1, 2, \dots$, until $3k + 1$ exceeds $3n - 3k - 1$. Using these two algorithms for each chain in the principal region, we generate the parameters of Eqs. (38) corresponding to the dihedral angles along all the edges which are branches of chains. It remains to do the same for all edges in the principal region which are not branches of chains. These edges connect the nodes of different but adjacent chains. Figure 15 shows two adjacent chains; the topmost chain, with m nodes, has the subscripts of its associated face angles labeled with the superscript 1; the bottom chain, with n nodes, has the subscripts of its associated face angles labeled with the superscript 2. Inspection of the diagram yields the algorithm for the dihedral angles along the edges which connect the nodes of the top and bottom chains:

CALL DIHD $\left(S_{3m-3k-2}^1, S_{3k+2}^1, S_{3k+3}^1, S_{3n-3k+2}^2, S_{3n-3k+3}^2, S_{3k-2}^2 \right)$,

for $k = 1, 2, \dots$, until $3k - 2$ exceeds $3n - 3k + 2$. Thus, to generate the parameters of the remaining equations, Eqs. (38), we use this algorithm for all pairs of adjacent chains in the principal region.

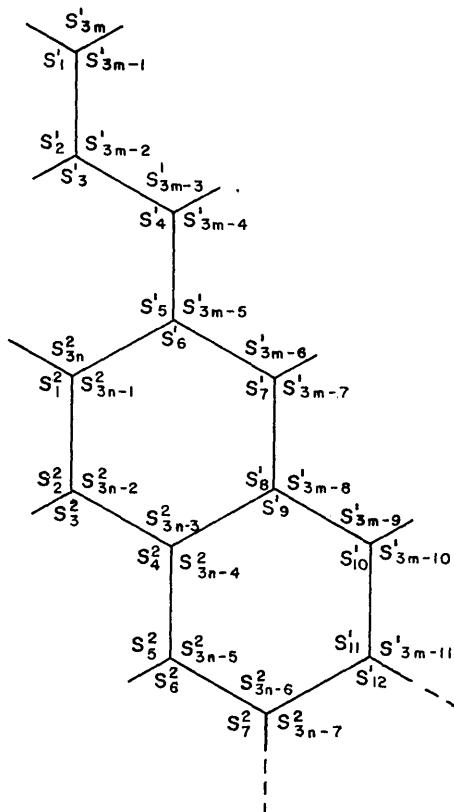


Fig. 15 - Two adjacent chains
having n and m nodes

We see that the chain method eliminates the tedious process of determining and listing the six-ordered face-angle subscripts for each call to the subroutine DIHD. If, instead, we label the p chains in the principal region from top to bottom as $C_{n_1}^1, \dots, C_{n_p}^p$, it suffices to read into the computer the ordered subscripts

$$S_1^1, \dots, S_{3n_1}^1, S_1^2, \dots, S_{3n_2}^2, \dots, S_1^p, \dots, S_{3n_p}^p.$$

Then the generation of all calls to DIHD can be handled automatically using the three algorithms just described. The linearized four-angle equations and sum-to-zero equations are also implemented by a streamlined method.

CONCLUSIONS

Trends Among Face Angles as Order of Polyhedra Increases

Table 2 indicates the computed values of some of the face angles for PH polyhedra up to 3242 faces. Figure 16 identifies the notation used. As usual, all symbols refer to excesses over $2\pi/3$ radians. The symbol ϵ labels the four identical face angles of each hexagon with twofold symmetry (Fig. 7a). The subscripts of the ϵ 's increase as the distance of the hexagon from the nearest pentagon increases; i.e., the nearest hexagon has ϵ_1 , the next nearest has ϵ_2 , etc. Figure 12 shows that these subscripts are the same as the hexagon type numbers. The symbol ω always refers to the positive excesses associated with the hexagons that have threefold symmetry (if there are any for a particular polyhedron). The symbol β refers to the face angle indicated in Fig. 16a.

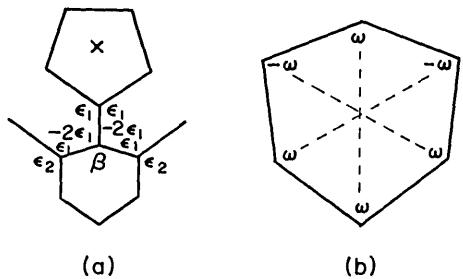


Fig. 16 - Illustration of the notation used for PH polyhedra up to 3242 faces: (a) location of face-angle excess labeled β and (b) hexagon with three-fold symmetry having positive face-angle excess ω

Inspection of Table 2 indicates that the excess ϵ_1 probably tends asymptotically toward 6° , and β tends toward 24° , as the orders of the polyhedra increase. This would mean that the three faces that meet at any pentagon vertex (the pentagon being one of the faces) tend to become coplanar. The same is true for the faces that meet at the vertex with associated excesses β , $-2\epsilon_1$, and $-2\epsilon_1$.

Although angles ϵ_2 through ϵ_9 are increasing with increasing polyhedron order, it is not possible to tell if they have an asymptotic limit.

The angle ω , although decreasing steadily, may not be approaching the value 0° . In fact, it probably cannot be ascertained that the decrease will continue for polyhedra with more than 3242 faces.

Approximate Diameters of Polyhedra

Table 3 lists the approximate diameters of the first 17 PH polyhedra as multiples of the edge length S . Actually, the diameter calculated in each case is that of a sphere

Table 2
Trends Among Angles

Angle Label	Angles for Various Numbers of Faces																	
	42	92	162	252	362	492	642	812	1002	1212	1442	1692	1962	2252	2562	2892	3242	
ϵ_1	1.72°	4.89°	5.57°	5.80°	5.94°	5.96°	5.97°	5.98°	5.987°	5.990°	5.992°	5.994°	5.995°	5.996°	5.997°	5.9976°		
ϵ_2	—	—	6.56°	7.12°	7.48°	7.70°	7.84°	7.92°	7.97°	8.01°	8.03°	8.046°	8.058°	8.066°	8.072°	8.077°	8.081°	
ϵ_3	—	—	—	—	7.30°	7.52°	7.75°	7.94°	8.09°	8.20°	8.27°	8.33°	8.369°	8.399°	8.422°	8.439°	8.452°	
ϵ_4	—	—	—	—	—	—	7.51°	7.62°	7.79°	7.95°	8.08°	8.20°	8.28°	8.352°	8.405°	8.447°	8.480°	
ϵ_5	—	—	—	—	—	—	—	—	7.60°	7.67°	7.78°	7.91°	8.04°	8.142°	8.232°	8.307°	8.369°	
ϵ_6	—	—	—	—	—	—	—	—	—	—	7.64°	7.68°	7.77°	7.880°	7.986°	8.084°	8.172°	
ϵ_7	—	—	—	—	—	—	—	—	—	—	—	—	—	—	7.677°	7.766°	7.853°	7.944°
ϵ_8	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	7.703°	7.759°	—
ϵ_9	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	7.687°
β	—	15.52°	20.53°	22.34°	23.10°	23.47°	23.66°	23.78°	23.84°	23.89°	23.92°	23.94°	23.95°	23.96°	23.97°	23.975°	23.980°	—
ω	—	15.52°	—	—	8.33°	—	—	5.67°	—	—	4.28°	—	—	3.44°	—	—	2.87°	—

having the same surface area as the polyhedron. To calculate the approximate surface area of one of these polyhedra, say one with N faces, all $N - 12$ hexagons are assumed to be regular; therefore, each has the approximate area $(3/2)\sqrt{3} S^2$, whereas each of the 12 pentagons has area $(5/4)S^2 \tan 54^\circ$. Hence, the approximate total surface area A is

$$A = \left[(N - 12) \frac{3}{2} \sqrt{3} + 15 \tan 54^\circ \right] S^2.$$

Equating this to the surface area of a sphere with diameter d , we have

$$d = \sqrt{\frac{1}{\pi} \left[(N - 12) \frac{3}{2} \sqrt{3} + 15 \tan 54^\circ \right]} S.$$

This formula generates Table 3.

Table 3
Approximate Diameters of PH Polyhedra

Number of Faces, N	Factor*	Number of Faces, N	Factor*
42	5.602	1212	31.61
92	8.528	1442	34.48
162	11.43	1692	37.36
252	14.32	1962	40.24
362	17.21	2252	43.12
492	20.09	2562	45.99
642	22.97	2892	48.87
812	25.85	3242	51.75
1002	28.73		

*For an edge of length S , the diameter equals factor multiplied by S .

Two Conjectures

The results obtained from the investigation of PH and other types of polyhedra suggest two theorems which may be valid for general polyhedra. The proposed theorems will be stated after some preliminary definitions are made.

Definition 1: An *element* of a polyhedron refers to a vertex, edge, or face.

Definition 2: We define *congruence* of polyhedron elements as follows.

A. Two vertices are congruent if the face angles and dihedral angles incident at one vertex are equal to the respective face angles and dihedral angles incident at the other vertex. It follows that two congruent vertices are either superposable or exact "mirror images" of each other.

B. Two edges are congruent if their lengths are equal and the dihedral angles formed at the edges are equal.

C. Two faces are congruent if the polygons comprising the faces are congruent in the sense of plane geometry. It follows that two congruent faces are either superposable or exact "mirror images" of each other.

We often examine the edges that intersect in a particular vertex, or the edges that bound a particular face. We can unify these two concepts by denoting the vertex or face

as a *primary* element and the edges that intersect in or bound the primary element as edges *incident* on the primary element. We speak of a *traverse* about a primary element as either a circuit along a closed path containing only a vertex and its incident edges or a circuit about the perimeter of a face. If two primary elements correspond under some mapping, then we say the set of edges incident on one of the primary elements corresponds in an *order-preserving* or *incidence-preserving* manner to the set of edges incident on the other primary element if, as we conduct traverses at the same "speed" about the primary elements, starting at a pair of corresponding edges, each pair of edges encountered simultaneously in the traverses corresponds under the mapping without interrupting the continuity of the traverses. The traverses may be either in the same sense (both clockwise or both counterclockwise) or in the opposite — i.e., mirror-image — sense (one clockwise and the other counterclockwise).

Definition 3: We say that two polyhedra P and Q are *isomorphic* (to each other) if and only if there exists a one-to-one map of P onto Q which maps vertices onto vertices, edges onto edges, and faces onto faces in an incidence-preserving manner, i.e., if (a) corresponding edges are bounded by corresponding pairs of vertices, (b) a pair of adjacent faces corresponds to a pair of adjacent faces, and the edge of intersection of one pair corresponds to the edge of intersection of the other pair, (c) all edges that are incident at one of two corresponding vertices correspond in an order-preserving manner to all the edges incident at the other vertex, and (d) all edges that bound one of two corresponding faces correspond in an order-preserving manner to all edges bounding the other face.

Definition 4: An *automorphism* of a polyhedron P is an isomorphism of P onto itself.

Definition 5: Two vertices of a polyhedron P are *equivalent* if there exists an automorphism of P which maps one vertex onto the other. Equivalence of edges (faces) is defined similarly.

Definition 6: Two polyhedra P and Q are said to be *duals* of each other (P is the dual of Q , and Q is the dual of P) if and only if there exists a one-one map of P onto Q , which maps vertices onto faces, faces onto vertices, and edges onto edges in an incidence-preserving manner; i.e., (a) an edge of $P(Q)$ is mapped onto an edge of $Q(P)$ if and only if the vertices bounding the edge of $P(Q)$ are mapped onto the faces intersecting in the edge of $Q(P)$, and (b) a vertex of $P(Q)$ is mapped onto a face of $Q(P)$ if and only if the edges incident at the vertex of $P(Q)$ are mapped in an order-preserving manner onto the edges which bound the face of $Q(P)$.

Statements (a) through (d) of definition 3 are not necessarily independent, nor are statements (a) and (b) of definition 6.

Theorem I: For any polyhedron P , there exists an isomorphic polyhedron Q , with the property that equivalent elements of Q are also congruent.

Theorem II: For any polyhedron P that is not self-dual, exactly one of the following statements is true: (a) there exists a unique isomorphic polyhedron Q , for which equivalent elements are congruent, and such that all edges have equal length, and (b) there exists a unique isomorphic polyhedron Q , for which equivalent elements are congruent, and such that all dihedral angles are equal. The statement that is false for P is true for the dual of P .

The truth or falsity of these theorems has not yet been established, because proofs have not yet been obtained. However, the authors have reasonable confidence that Theorem I is true. The veracity of Theorem II is less certain.

NUMERICAL RESULTS

Corresponding to each PH polyhedron with no more than 3242 faces are five entities which describe its solution and the values of some relevant parameters sufficient for its physical construction. These are (a) a drawing including at least one principal region, in which the hexagon types are numbered, pentagons are labeled with x's, and the face-angle locations are identified by subscripts in the principal region, (b) the values, in degrees, of the initial guess used for the face-angle excesses, (c) the solution values of all face-angle excesses, (d) the lengths of all hexagon diagonals, assuming all edges of the polyhedron have unit length, and (e) the values of all dihedral angles. Descriptions follow for all entities.

Drawings — The drawings comprise Fig. 17. Each drawing is similar to Fig. 12 (cf. description accompanying Fig. 12), although some show only the principal region. Pentagons are indicated by x's in their centers. Hexagon types are indicated by numbers in their centers, beginning with the exterior diagonal row and proceeding inward to the center of the principal region. Within a diagonal row the hexagon types are numbered in ascending order beginning at the vertical boundary and terminating at the diagonal boundary. The vertex angles of the hexagons and pentagon (i.e., the polyhedron face angles) in the principal region are labeled with subscripts in ascending order according to the convention (a) the off-axis vertex angles of the hexagons with twofold symmetry, increasing according to increasing hexagon number; (b) the vertex angle of the hexagon with threefold symmetry (if one exists), the particular vertex chosen lying on the vertical boundary; (c) the vertex angles of hexagons with onefold symmetry, four consecutive subscripts per hexagon, increasing within a hexagon in a direction toward the interior of the principal region, increasing between hexagons according to increasing hexagon number; (d) the vertex angles of hexagons with no symmetry, six consecutive subscripts per hexagon, increasing within a hexagon in a counterclockwise direction beginning at the top, increasing between hexagons according to increasing hexagon number; (e) the on-axis vertex angles of hexagons with twofold symmetry, increasing according to increasing hexagon number; (f) the vertex angle of the hexagon with threefold symmetry (if one exists), the particular vertex chosen lying on the diagonal boundary; and (g) the pentagon vertex angle (excess = -12°).

Initial Values and Solution Values for Face Angles — The values, in degrees, of the excesses of the face angles over 120°, obtained after convergence of an iterative Newton-Raphson process using the indicated initial guess are in Appendix B. The number of required iterations is also shown. Each excess is identified by an index corresponding to the subscript of equal numerical value shown in the drawing.

Hexagon Diagonals — The lengths of the nine diagonals of each equilateral hexagon are given in Appendix B in three rows and three columns, based on sides of unit length. The first two columns give the lengths of the short diagonals — those connecting vertices at the noncommon endpoints of a pair of adjacent sides — and the third column gives the length of the long diagonals. The length of a diagonal connecting vertices associated with face-angle subscripts i and j is denoted by $L(i, j)$. No ambiguity results in labeling diagonals of symmetric hexagons in this manner, since the short and long diagonals are separate. The hexagon diagonals are printed in the order (a) diagonals of hexagons with twofold symmetry, (b) diagonals of the hexagon with threefold symmetry (if any), (c) diagonals of hexagons with onefold symmetry, and (d) diagonals of hexagons with no symmetry. Within these categories, order is determined according to hexagon type number.

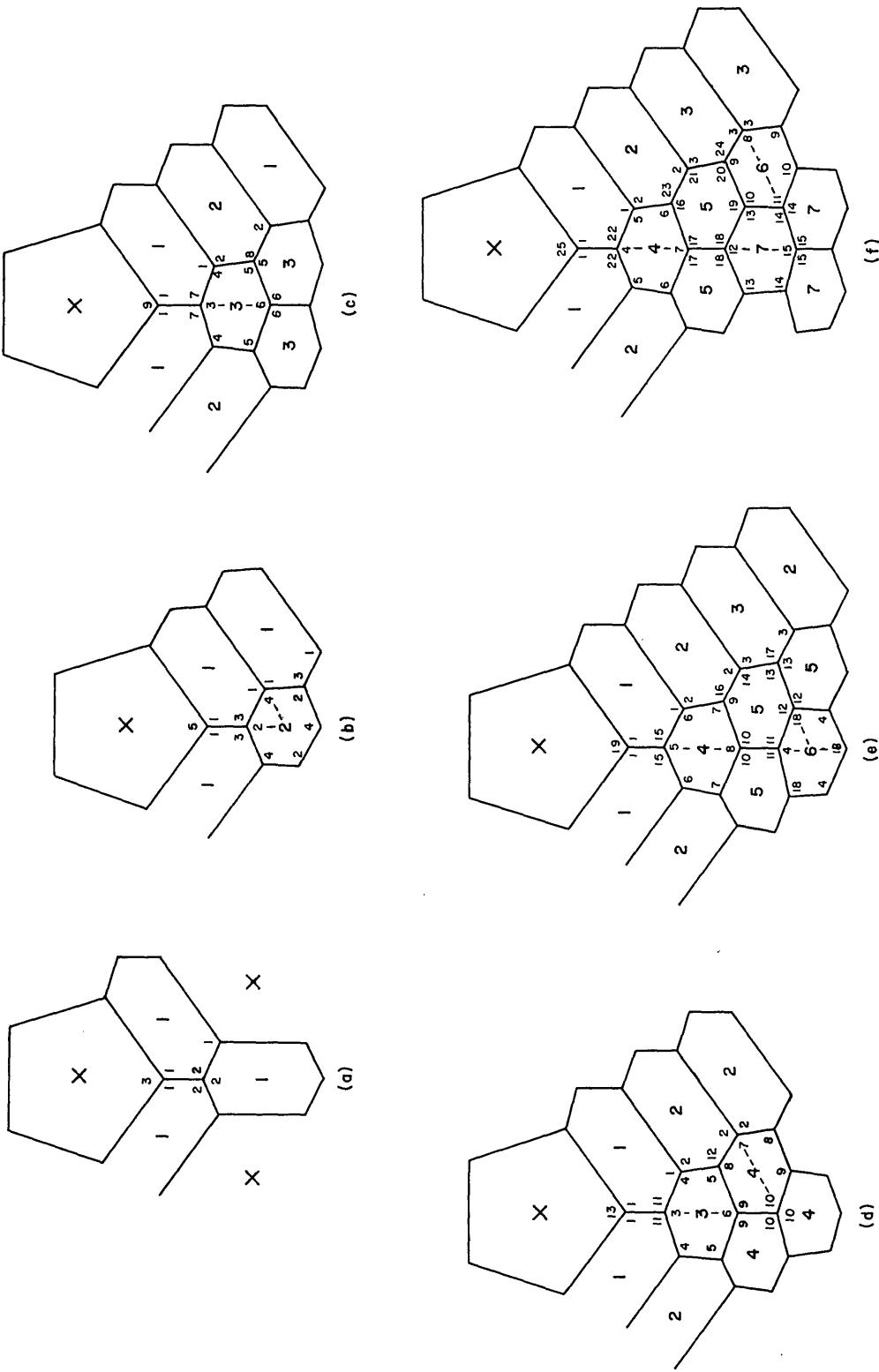
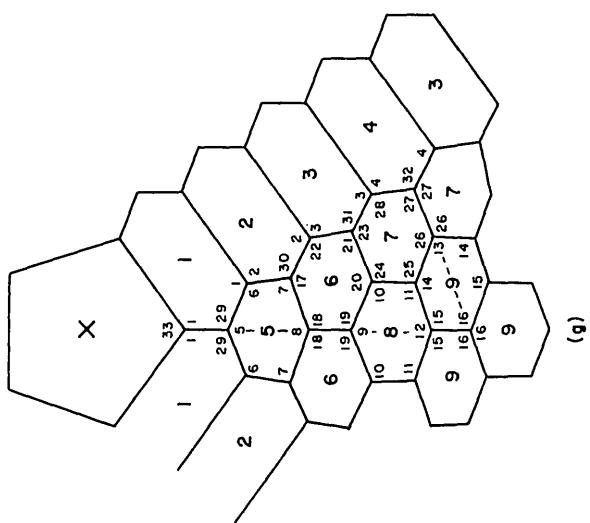
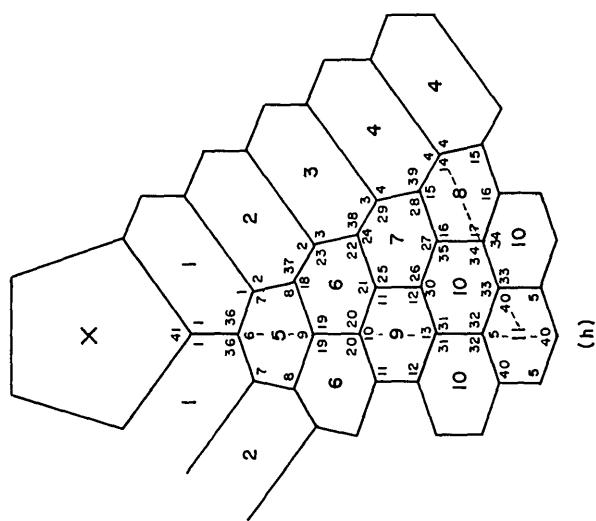
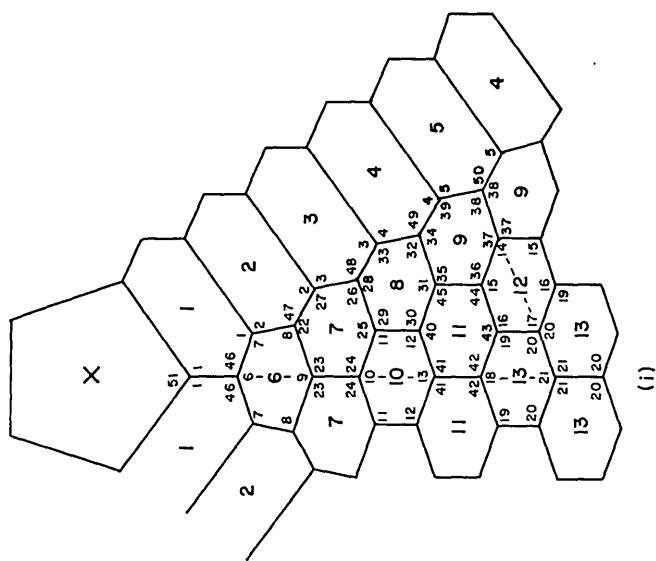


Fig. 17 - PH polyhedra up to 3242 faces identifying the important parts: (a) 42-faced polyhedron, (b) 92-faced polyhedron, (c) 162-faced polyhedron, (d) 252-faced polyhedron, (e) 362-faced polyhedron, (f) 492-faced polyhedron, (g) 642-faced polyhedron, (h) 812-faced polyhedron, (i) 1002-faced polyhedron, (j) 1212-faced polyhedron, (k) 1442-faced polyhedron, (l) 1692-faced polyhedron, (m) 1962-faced polyhedron, (n) 2252-faced polyhedron, (o) 2562-faced polyhedron, (p) 2892-faced polyhedron, and (q) 3242-faced polyhedron



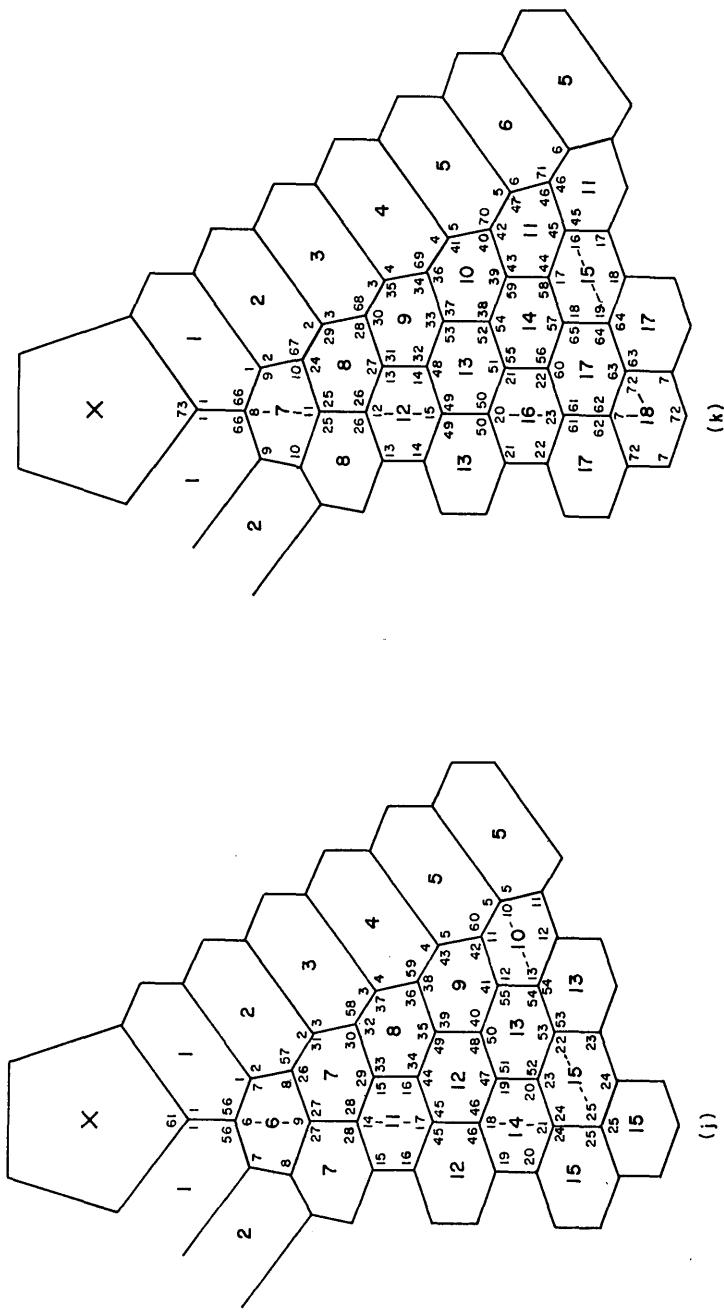
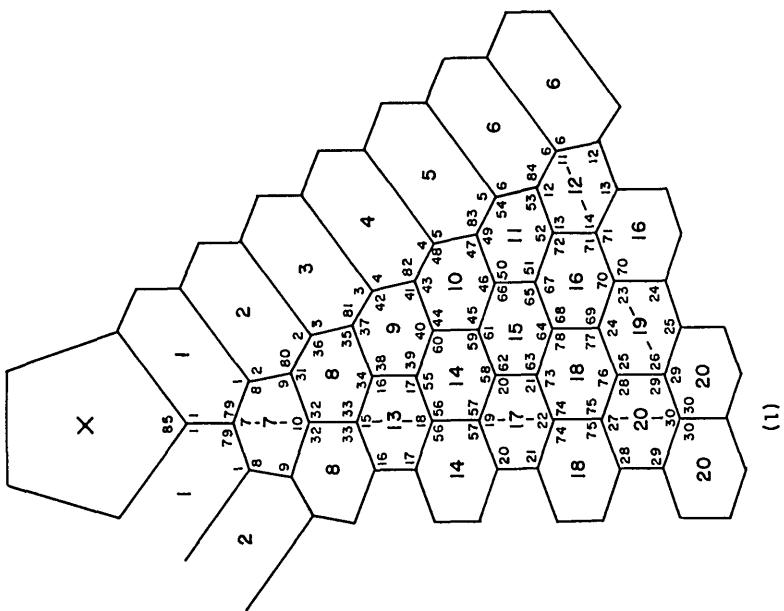
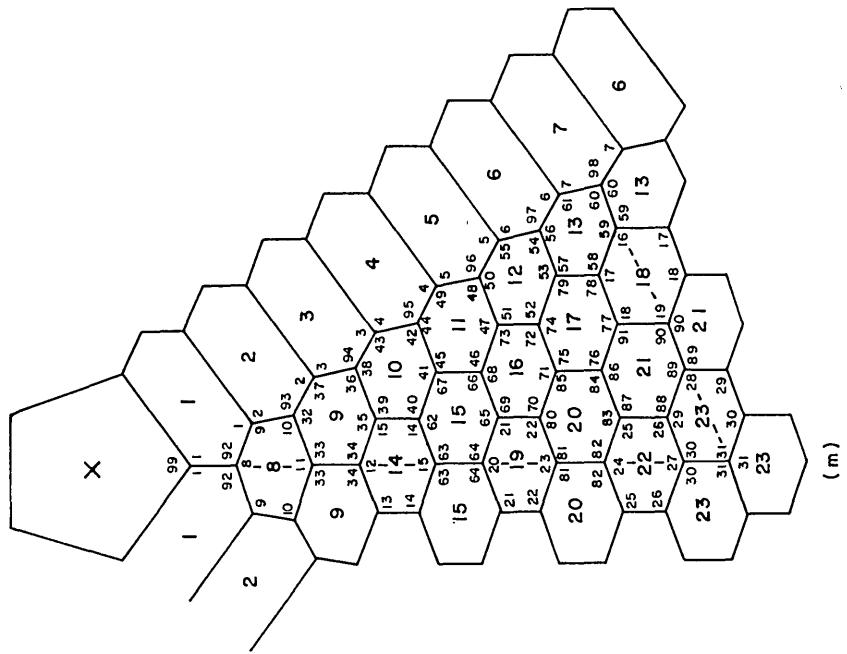


Fig. 17 (Continued) - PH polyhedra up to 3242 faces identifying the important parts: (a) 42-faced polyhedron, (b) 92-faced polyhedron, (c) 162-faced polyhedron, (d) 252-faced polyhedron, (e) 362-faced polyhedron, (f) 492-faced polyhedron, (g) 642-faced polyhedron, (h) 812-faced polyhedron, (i) 1002-faced polyhedron, (j) 1212-faced polyhedron, (k) 1442-faced polyhedron, (l) 1692-faced polyhedron, (m) 1962-faced polyhedron, (n) 2252-faced polyhedron, (o) 2562-faced polyhedron, (p) 2892-faced polyhedron, and (q) 3242-faced polyhedron



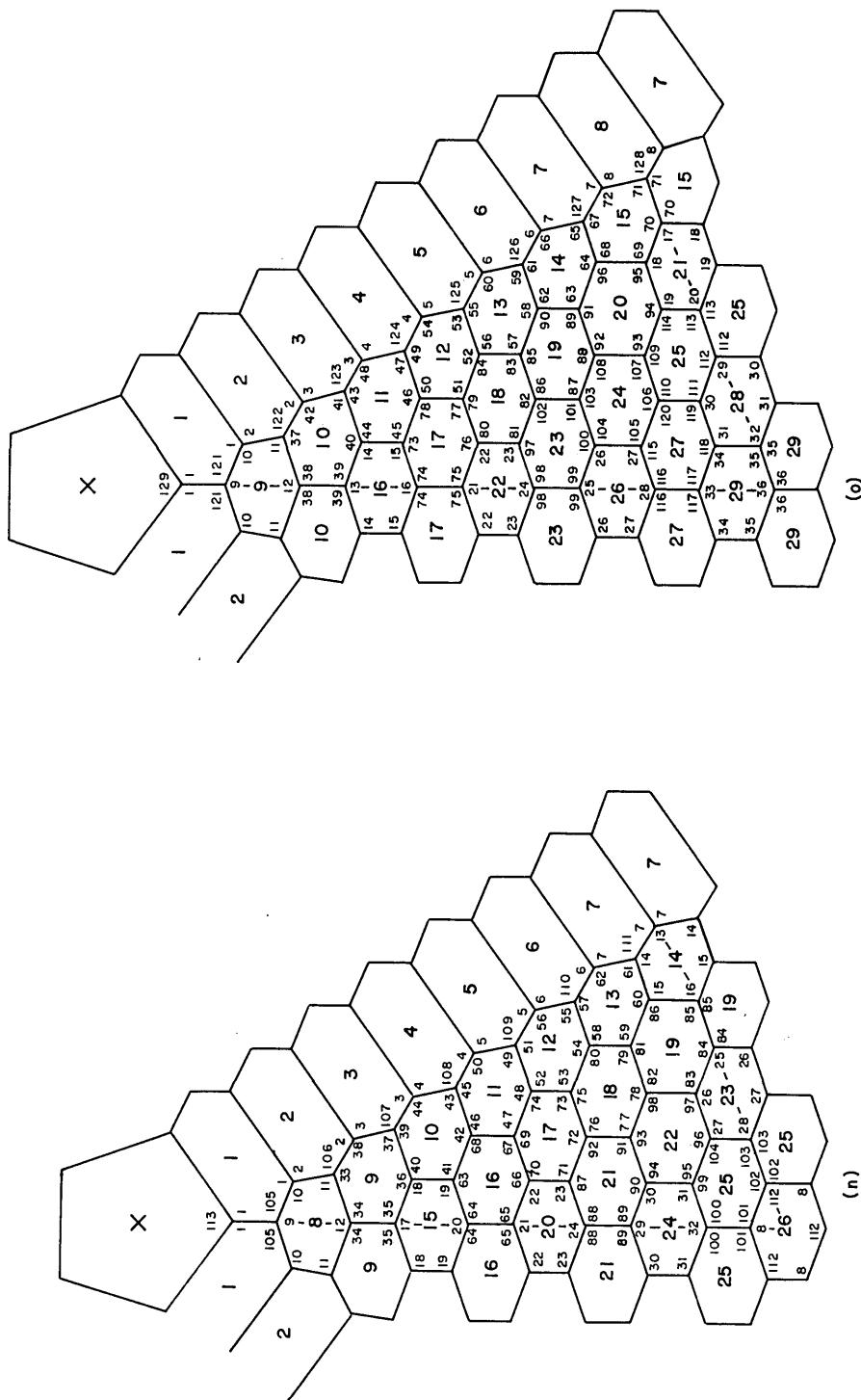


Fig. 17 (Continued) - PH polyhedra up to 3242 faces identifying the important parts: (a) 42-faced polyhedron, (b) 92-faced polyhedron, (c) 162-faced polyhedron, (d) 252-faced polyhedron, (e) 362-faced polyhedron, (f) 492-faced polyhedron, (g) 642-faced polyhedron, (h) 812-faced polyhedron, (i) 1002-faced polyhedron, (j) 1212-faced polyhedron, (k) 1442-faced polyhedron, (l) 1692-faced polyhedron, (m) 1962-faced polyhedron, (n) 2252-faced polyhedron, (o) 2562-faced polyhedron, (p) 2892-faced polyhedron, and (q) 3242-faced polyhedron

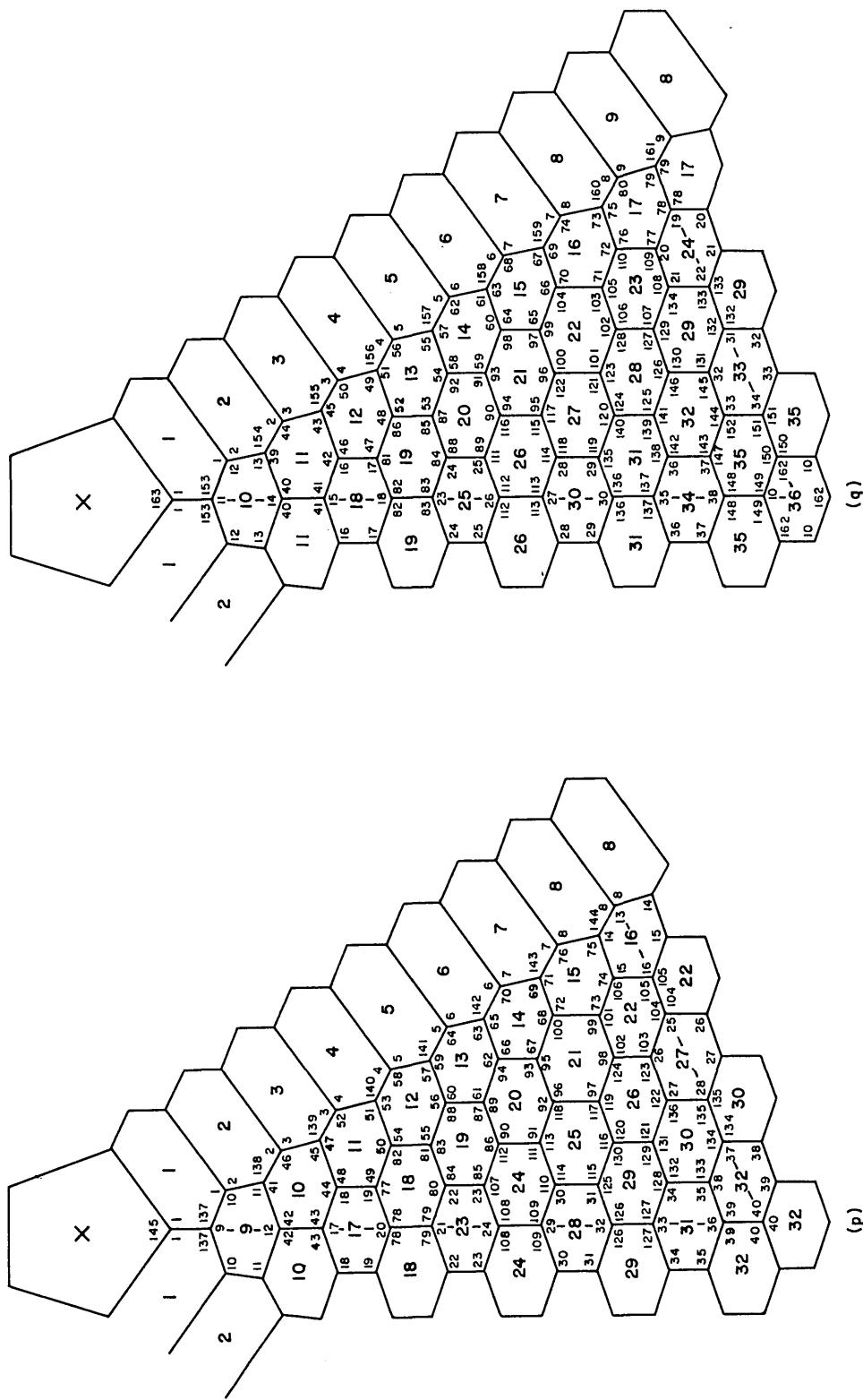


Fig. 17 (Continued) - PH polyhedra up to 3242 faces identifying the important parts: (a) 42-faced polyhedron, (b) 92-faced Polyhedron, (c) 162-faced polyhedron, (d) 252-faced polyhedron, (e) 362-faced polyhedron, (f) 492-faced polyhedron, (g) 642-faced polyhedron, (h) 812-faced polyhedron, (i) 1002-faced polyhedron, (j) 1212-faced polyhedron, (k) 1442-faced polyhedron, (l) 1692-faced polyhedron, (m) 1962-faced polyhedron, (n) 2252-faced polyhedron, (o) 2562-faced polyhedron, (p) 2892-faced polyhedron, and (q) 3242-faced polyhedron

Dihedral Angles — Each dihedral angle in Appendix B is identified by an ordered triplet of integers, each integer being a face-angle subscript. In Fig. 18a the dihedral angle of interest is formed by the faces that intersect in the edge adjacent to face angles (excesses) α_i and α_k . Figure 18b shows the spherical representation of this. We denote this dihedral angle by $\mu(j, i, k)$, the 1st and 3rd integers being the subscripts of the adjacent face angles. We have

$$\mu(j, i, k) = \arccos \left[\frac{\cos \left(\frac{2\pi}{3} + \alpha_i \right) - \cos \left(\frac{2\pi}{3} + \alpha_j \right) \cos \left(\frac{2\pi}{3} + \alpha_k \right)}{\sin \left(\frac{2\pi}{3} + \alpha_j \right) \sin \left(\frac{2\pi}{3} + \alpha_k \right)} \right]$$

This angle is identified in the computer printout by the ordered triplet j, i, k .

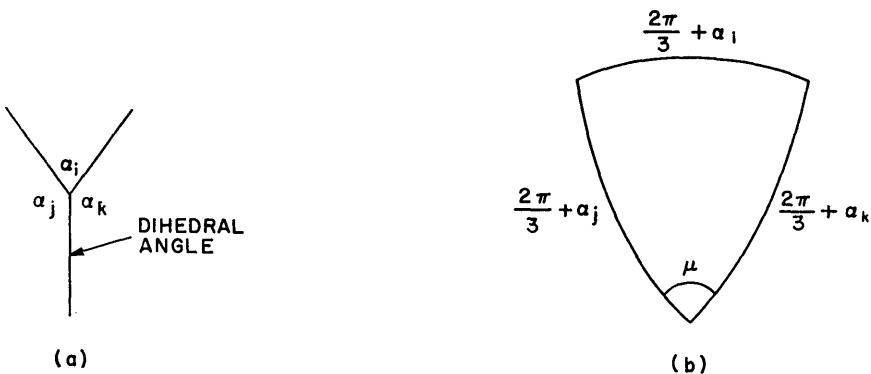


Fig. 18 - Identification of a typical dihedral angle:
(a) actual view and (b) spherical representation

Appendix A

PROGRAM CONTENT AND USE

Table A1 is a listing in CDC 3800 FORTRAN of the main program and subroutines used for PH polyhedra with more than 2252 faces, in which the system of equations, Eqs. (11), is solved in single precision. The technique employed for solution is that of Gaussian elimination by means of the CDC COOP subroutine F2UTEX GAUSS2, written by C. B. Bailey.

We now describe the data cards used for the solution of a PH polyhedron. Only one PH polyhedron may be solved in a given run, since different polyhedra require different sets of cards.

We first remark that we label the face-angle subscripts in accordance with the convention of Fig. 17, illustrated in Fig. 12 for the 1002-faced PH polyhedron. In the first part of the last section of the report, the angles belonging to categories (a) through (d) are called *basic* angles; the other angles (categories (e) through (g)) are called *auxiliary* angles.

Five types of data cards are used. They are, in sequence, (a) Type 1: the title card, (b) Type 2: the general information card, (c) Type 3: the chain information card(s), (d) Type 4: the hexagon information card(s), and (e) Type 5: the initial value card(s). Their descriptions follow.

Type 1: The entire type-1 card is available for titling, labeling, or other miscellaneous information. Its contents are reproduced in the first line of the program output.

Type 2: The type-2 card is read into storage under FORMAT (8I3). It has the following information:

- Cols. 1-3: The number of hexagons with twofold symmetry,
- Cols. 4-6: The number of hexagons with threefold symmetry (0 or 1),
- Cols. 7-9: The number of basic angles,
- Cols. 10-12: The number of equations (equals the total number of face angles, both basic and auxiliary),
- Cols. 13-15: The mode number. If 0 or blank, one set of iterations will be performed for a specified set of initial values. Otherwise, repeated sets of iterations will be performed, one set for each of a series of specified constant initial values for the basic angles,
- Cols. 16-18: The number of constant initial values to be used (irrelevant if Cols. 13-15 are 0 or blank),
- Cols. 19-21: If nonblank, the lengths of the hexagon diagonals will be computed and printed; suppressed if 0 or blank, and
- Cols. 22-24: If nonblank, values of the dihedral angles will be computed and printed; suppressed if 0 or blank.

Type 3: The type-3 cards contain the ordered subscripts of the chains (cf. description of the chain method in the section: "Computational Techniques.") Each card is read into storage under FORMAT (24I3); as many consecutive cards as necessary may be used. The data are punched in the sequence $S_1^1, \dots, S_{3n_1}^1$, blank field, $S_1^2, \dots, S_{3n_2}^2$, blank field, \dots , blank field, $S_1^p, \dots, S_{3n_p}^p$, two blank fields.

Type 4: The type-4 cards contain the face-angle subscript of smallest numerical value for each of the hexagons with onefold symmetry, followed by the face-angle subscript of smallest numerical value for each of the hexagons with no symmetry. Each card is read into storage under FORMAT (24I3); as many consecutive cards as necessary may be used. The representative subscripts for the onefold symmetric hexagons (if any) are punched first, then a blank field, then the representative subscripts for the asymmetric hexagons (if any), followed by another blank field.

Type 5: The type-5 cards are read in under FORMAT (10E8). Their contents depend on the contents of Columns 13 through 18 of the general information card. If the mode number (13-15) is 0 or blank, these cards contain, in sequence, the initial values of all the basic angles. The program then performs one set of Newton-Raphson iterations using these initial values, and it terminates when convergence is obtained for all face angles. If the mode number is nonzero and nonblank, these cards contain a series of constant initial face-angle values equal in number to that specified in Columns 16 through 18 of the general information card. The program performs one set of Newton-Raphson iterations for each of the constant initial values, the latter being used to initialize all basic angles.

Figure A1 indicates the data-card listing corresponding to the 2562-faced PH polyhedron (see also Fig. 17o). As indicated, there are eight hexagons with twofold symmetry, 120 basic angles, and nine auxiliary angles (thus 129 equations). The computation and printout of the hexagon diagonals and the dihedral angles will be included. The program will first set the values of all basic angles equal to 0.01 and perform the Newton-Raphson process until convergence to a solution has occurred. Then the values of all basic angles will be set equal to 0.1, and the Newton-Raphson process will again be used to converge to a solution. The two solutions obtained are the same, as mentioned in the section "Computational Techniques."

Fig. A1 - Data card listing for a 2562-faced PH polyhedron

Table A1
Program Listing

PROGRAM PLYHDRN	5
COMMON /TFN1/A(169,170),/TFN2/AGR(169),SS(169),CC(169),/EQU/NE,NQ	10
1/TWPI3/TPT /DIAG/DPR,AG(169)	11
TYPE DOUBLE SS,CC,TPT,PI,AG,AGR,DPR	15
DATA (TOL=1E-5), (A1=1E-20), (PI=3.141592653589793238462643D),	20
1(IF2=46H(14H INITIAL GUESS// (4H AG(13,3H) =F10.2/)))	21
DIMENSION X(169),IF2(6),AG1(50),ICHN(10,100),ND3(10),	25
IISTR(24),ISHX(100),KTITLE(10)	26
READ 70, KTITLE	30
PRINT 71, KTITLE	35
READ 15, NS2,NS3,NB,NE,MODE,NIV,IHD,IDA	40
IC = 1 \$ JC = IFLG = 0	45
41 READ 42, ISTR	50
DO 43 I=1,24	55
IF(ISTR(I))44,45,44	60
44 IFLG = 0	65
JC = JC+1	70
ICHN(IC,JC) = ISTR(I)	75
43 CONTINUE	80
GO TO 41	85
45 IF(IFLG-1)46,47,46	90
46 IFLG = 1	95
ND3(IC) = JC	100
IC = IC+1 \$ JC = 0	105
GO TO 43	110
47 NCHN = IC-1	115
JC = IFLG = 0	120
48 READ 42, ISTR	125
DO 49 I=1,24	130
IF(ISTR(I))60,61,60	135
61 IF(IFLG)62,63,62	140
63 NTO = JC	145
IFLG = 1	150
GO TO 49	155
60 JC = JC+1	160
ISHX(JC) = ISTR(I)	165
49 CONTINUE	170
GO TO 48	175
62 NT1 = JC-NTO	180
IF(MODE,NE,0)GO TO 30	185
NIV = 1	190
READ 1, (AG(I),I=1,NB)	195
GO TO 31	200
30 READ 1, (AG(I),I=1,NIV)	205
31 DPR = 180./PI	210
TPT = 2.*PT/3.	215
TOLR = TOL/DPR	220
ENCODE(8,2,IF2(3)),IF2(3),NE,TF2(3)	225
DO 90 ICNT=1,NIV	230
IF(MODE)33,34,33	235
33 DO 35 I=1,NB	240
35 AG(I) = AG1(ICNT)	245
34 ITER = 1	250
C	255
C * * * * * INITIAL VALUES * * * * *	
C	
DO 21 I=1,NS2	260
IC = NTO+1	265
21 AG(IC) = -2.*AG(I)	270

Table A1 (Continued)
Program Listing

IF(NS3)22,23,22	275
22 IC = NS2+1 \$ JC = IC+NR	280
AGR(JC) = -AGR(IC)	285
23 AG(NE) = -12.0	290
C	
IF(MODE)82,83,8?	2922
82 PRINT 74, NB,AG(1)	2924
I = NB+1	2926
PRINT 75, (J,AG(J),J=I,NE)	2928
GO TO 11	2930
83 PRINT 1F2, (I,AG(I),I=1,NE)	295
PRINT 101	297
11 IF(MODE)36,37,36	300
37 PRINT 103, ITER	302
PRINT 7	305
36 DO 3 I=1,NE	310
AGR(I) = AG(I)/DPR	315
DO 3 J=1,NE	320
3 A(I,J) = 0.	325
NO = 0	330
CALL TRIGFN(NE)	335
C	
C * * * * * DETERMINATION OF MATRIX COEFFICIENTS * * * * *	
C	
DO 52 IC=1,NCHN	340
K3 = 3 \$ ICM1 = IC-1	345
54 MK3 = ND3(IC)-K3	350
KMK3 = K3-MK3	355
IF(ICM1)55,56,55	360
55 IF(KMK3-4)57,57,52	365
57 JC = ND3(ICM1)-K3-2	370
CALL DIHDT(ICHN(ICM1,JC),ICHN(ICM1,K3+2),ICHN(ICM1,K3+3),ICHN(IC, 1MK3+2),ICHN(IC,MK3+3),ICHN(IC,K3-2))	375
56 IF(KMK3-1)59,59,51	376
51 IF(ICM1)50,52,50	380
59 CALL DIHDT(ICHN(IC,MK3+3),ICHN(IC,K3-2),ICHN(IC,MK3+2),ICHN(IC, 1MK3+1),ICHN(IC,K3-1),ICHN(IC,K3))	385
53 CALL DIHD(ICHN(IC,K3-1),ICHN(IC,K3),ICHN(IC,MK3+1),ICHN(IC,MK3), 1ICHN(IC,K3+1),ICHN(IC,MK3-1))	390
50 K3 = K3+3	391
GO TO 54	395
52 CONTINUE	400
IF(NT0)66,67,66	401
66 DO 64 IC=1,NT0	405
64 CALL HEXAGON(1,ISHX(IC))	410
67 IF(NT1)68,69,68	415
68 DO 65 IC=1,NT1	420
JC = NT0+IC	425
65 CALL HEXAGON(1,ISHX(JC))	430
69 DO 20 I=1,NS2	435
J = NS2+I \$ A(J,I) = ? \$ A(J,J) = 1.	440
20 A(J,NE+1) = -2.*AGR(I)-AGR(J)	445
IF(NS3)24,25,24	450
24 IC = NS2+1 \$ JC = IC+NR	455
A(JC,IC) = A(JC,JC) = 1. \$ A(JC,NE+1) = -AGR(IC)-AGR(JC)	460
25 A(NE,NE) = 1. \$ A(NE,NE+1) = 0.	465
C	
CALL GAUSSP(NE,1,A1,A,X,K3)	470
	475
	480
	485
	500

Table A1 (Continued)
Program Listing

```

IF(K3.EQ.2)GO TO 5      505
DO 6 I=1,NE              510
6 AG(I) = AG(I)+DPR*X(I) 515
IF(MODE)38,39,38          520
38 DO 8 I=1,NE            530
  XBSS = ABSF(X(I))       535
  IF(XBSS.GE.TOLR)GO TO 9 540
8 CONTINUE                545
  IF(MODE)40,10,40          550
40 PRINT 101               552
  PRINT 102, ITER           553
  PRINT 7                  555
39 NETD = (NE+2)/3        5625
DO 80 I=1,NFTD            5630
  J = I+NETD $ K = J+NFTD 5635
  IF(K.GT.NE)GO TO 81       5640
  PRINT 72, I,AG(I),J,AG(J),K,AG(K)
  GO TO 80                 5645
81 PRINT 73, I,AG(I),J,AG(J)
80 CONTINUE                5650
  IF(MODE)10,38,10          5655
9 ITER = ITER+1            5660
  IF(ITER.LE.30)GO TO 11     5665
  PRINT 12
  GO TO 90                 5670
5 PRINT 13                 5675
  GO TO 90                 5680
10 IF(IHD)91,110,91         5685
91 PRINT 100               5690
DO 92 I=1,NS2              5692
  IC = NB+I
92 CALL HEXDIAG(IC,I,I,IC,I,I) 5693
  IF(NS3)93,94,93          5694
93 IC = NS2+1 $ JC = IC+NB 5694
  CALL HEXDIAG(IC,JC,IC,JC,IC,JC)
94 IF(NT0)95,96,95          5694
95 DO 97 I=1,NT0             5695
  IC = ISHX(I)
97 CALL HEXDIAG(IC,IC+1,IC+2,IC+3,IC+2,IC+1)
96 IF(NT1)98,110,98          5695
98 DO 99 I=1,NT1             5696
  J = NT0+I
  IC = ISHX(J)
99 CALL HEXDIAG(IC,IC+1,IC+2,IC+3,IC+4,IC+5)
110 IF(IDA)111,90,111        5696
111 IRFS = 0                 5697
DO 118 I=1,NF               5697
118 AGR(I) = AG(I)/DPR      5698
  CALL TRIGFN(NE)
PRINT 104                  5699
DO 112 I=1,NCHN             5700
  K3 = 3
117 MK3 = ND3(I)-K3          5701
  KMK3 = K3-MK3
  IF(KMK3-4)113,113,112      5702
113 CALL DHANGLE(0,IRFS,ICHNT(I,MK3+3),ICHNT(I,K3-2),ICHNT(I,MK3+2)) 5703
  IF(K3.EQ.3)GO TO 114        5704
  CALL DHANGLE(0,IRFS,ICHNT(I,K3-2),ICHNT(I,MK3+2),ICHNT(I,MK3+3))
114 IF(KMK3-2)115,115,116      5705

```

Table A1 (Continued)
Program Listing

115 CALL DHANGLE(0,IRES,ICHN(I,K3-2),ICHN(I,MK3+3),ICHN(I,MK3+2))	5984
116 K3 = K3+3	5985
GO TO 117	5986
112 CONTINUE	5987
CALL DHANGLE(1,IRES,0,0,0)	5988
90 PRINT 32	599
42 FORMAT(24I3)	600
15 FORMAT(8I3)	605
1 FORMAT(10E8)	610
2 FORMAT(A4,I3,RI)	615
101 FORMAT(9(/),* FACE ANGLFS -- EXPRESSED AS EXCESS OVER 120 1DEG*)	616
102 FORMAT(//I3,20H ITERATIONS REQUIRED//)	617
103 FORMAT(7(/),1H ITERATION I3//)	619
100 FORMAT(9(/),18H HEXAGON DIAGONALS// 17X,15H SHORT DIAGONALS 25X,14HL 1ONG DIAGONALS//)	620
7 FORMAT(3(6H INDIX9X,11HEXCESS(DEF)12X)//)	621
12 FORMAT(///* NO CONVERGENCE AFTER 30 ITERATIONS*)	625
13 FORMAT(///* EQUATIONS ARE INCONSISTENT OR DEPENDENT*)	630
104 FORMAT(9(/),36H DIHEDRAL ANGLES -- EXPRESSED IN DEG///3(16H ORDERED 1D TRIPLET6X,1CHDIHD ANGLE7X)//)	635
32 FORMAT(1H)	637
74 FORMAT(14H INITIAL GUESS//15H AG(1) THRU AG(I3,3H) =E10.2)	640
75 FORMAT(4H AG(I3,3H) =E10.2)	642
70 FORMAT(10A8)	643
71 FORMAT(1H 10A8//)	645
72 FORMAT(3(I6,E20.6,12X))	650
73 FORMAT(2(I6,E20.6,12X))	652
END	653
SUBROUTINE TRIGFN(NE)	TF 5
COMMON /TFN1/A(169,170) /TFN2/AGR(169),SS(169),CC(169) /TWPI3/TPT	TF 10
TYPE DOUBLE SS,CC,TPT,AGR,AGR	TF 15
DO 1 I=1,NE	TF 20
AGR = TPT+AGR(I)	TF 25
SS(I) = DSIN(AGR)	TF 30
I CCT(I) = DCOS(AGR)	TF 35
END	TF 40
SUBROUTINE DIHD(I1,I2,I3,I4,I5,I6)	LZ 5
COMMON /TFN1/A(169,170) /TFN2/AGR(169),SS(169),CC(169) /EQU/NE,NQ	LZ 10
TYPE DOUBLE SS,CC,C123,C645,AGR	LZ 15
NQ = NQ+1	LZ 17
A(NQ,I1) = -SS(I1)*SS(I4)*SS(I5)	LZ 20
C123 = CC(I1)-CC(I2)*CC(I3)	LZ 25
C645 = CC(I6)-CC(I4)*CC(I5)	LZ 30
A(NQ,I2) = A(NQ,I2)+	LZ 32
I SS(I2)*CC(I3)*SS(I4)*SS(I5)-C645*CC(I2)*SS(I3)	LZ 35
A(NQ,I3) = A(NQ,I3)+	LZ 37
I CC(I2)*SS(I3)*SS(I4)*SS(I5)-C645*SS(I2)*CC(I3)	LZ 40
A(NQ,I4) = A(NQ,I4)+	LZ 42
I C123*CC(I4)*SS(I5)-SS(I2)*SS(I3)*SS(I4)*CC(I5)	LZ 45
A(NQ,I5) = A(NQ,I5)+	LZ 47
I C123*SS(I4)*CC(I5)-SS(I2)*SS(I3)*CC(I4)*SS(I5)	LZ 50
A(NQ,I6) = A(NQ,I6)+	LZ 52
I SS(I2)*SS(I3)*SS(I6)	LZ 55
A(NQ,NE+1) = -C123*SS(I4)*SS(I5)+C645*SS(I2)*SS(I3)	LZ 60
END	LZ 65
SUBROUTINE HEXAGON(IT,IS)	HX 5
ITSP1 = IS+1 \$ ITSP2 = IS+2 \$ ITSP3 = IS+3	HX 10
IF (IT)1,2,1	HX 15

Table A1 (Continued)
Program Listing

```

2 CALL FOURANG(IS,ISP1,ISP2,ISP3) HX 20
CALL SMTOZ(IS,ISP1,ISP1,ISP2,ISP2,ISP3) HX 25
RETURN HX 30
1 ISP4 = IS+4 $ ISP5 = IS+5 HX 35
CALL FOURANG(IS,ISP5,ISP2,ISP3) HX 40
CALL FOURANG(ISP1,IS,ISP3,ISP4) HX 45
CALL SMTOZ(IS,ISP1,ISP2,ISP3,ISP4,ISP5) HX 50
END HX 55
SUBROUTINE FOURANG(J1,J2,J3,J4)
COMMON /TFN1/A(169,170) /TFN2/AGR(169),SS(169),CC(169) /EQU/NE,NQ FA 10
TYPE DOUBLE SS,CC,SC12,SC34,AGR
NQ = NQ+1 FA 15
FA 17
SC12 = SS(J1)*CC(J2)+CC(J1)*SS(J2) FA 20
SC34 = SS(J3)*CC(J4)+CC(J3)*SS(J4) FA 25
A(NQ,J1) = -SS(J1)+SC12 FA 30
A(NQ,J2) = A(NQ,J2) FA 32
1 -SS(J2)+SC12 FA 35
A(NQ,J3) = A(NQ,J3)+ FA 37
1 SS(J3)-SC34 FA 40
A(NQ,J4) = A(NQ,J4)+ FA 42
1 SS(J4)-SC34 FA 45
A(NQ,NE+1)=-(CC(J1)+CC(J2)-CC(J1)*CC(J2)+SS(J1)*SS(J2))+CC(J3)+CC(J4)-CC(J3)*CC(J4)+SS(J3)*SS(J4) FA 50
1J4)-CC(J3)*CC(J4)+SS(J3)*SS(J4) FA 50
END FA 55
SUBROUTINE SMTOZ(I1,I2,I3,I4,I5,I6)
COMMON /TFN1/A(169,170) /TFN2/AGR(169),SS(169),CC(169) /EQU/NE,NQ SZ 10
TYPE DOUBLE AGR,SS,CC
NQ = NQ+1 SZ 15
SZ 17
A(NQ,I1) = 1. $ A(NQ,I2) = A(NQ,I2)+1. SZ 20
A(NQ,I3) = A(NQ,I3)+1. $ A(NQ,I4) = A(NQ,I4)+1. SZ 25
A(NQ,I5) = A(NQ,I5)+1. $ A(NQ,I6) = A(NQ,I6)+1. SZ 30
A(NQ,NE+1) = -AGR(I1)-AGR(I2)-AGR(I3)-AGR(I4)-AGR(I5)-AGR(I6) SZ 35
END SZ 40
SUBROUTINE HEXDIAG(I1,I2,I3,I4,I5,I6)
COMMON /DIAG/DPR,AG(169) HD 5
TYPE DOUBLE DPR,E(6), AG,A(6),COSA(6) HD 10
DIMENSION EL(9) HD 15
HD 17
F(1) = AG(I1) $ F(2) = AG(I2) $ F(3) = AG(I3) HD 20
E(4) = AG(I4) $ E(5) = AG(I5) $ E(6) = AG(I6) HD 25
DO 1 I=1,6 HD 30
A(I) = (E(I)+120.D)/DPR HD 35
COSA(I) = DCOS(A(I)) HD 40
1 EL(I) = DSQRT(2.*(1.-COSA(I))) HD 45
DO 2 I=1,3 HD 50
2 EL(I+6) = DSQRT(3.-2.*((COSA(I)+COSA(I+1))-DCOS(A(I)+A(I+1)))) HD 55
PRINT 3, I2,I6,EL(1),I3,I5,EL(4),I3,I6,EL(7),I1,I3,EL(2),I4,I6, HD 60
1EL(5),I1,I4,EL(8),I2,I4,EL(3),I1,I5,EL(6),I2,I5,EL(9) HD 61
3 FORMAT(3(3H L(I3,1H,I3,3H) =F9.6, 4X)//) HD 65
END HD 70
SUBROUTINE DHANGLE(MODE,IR,J,I,K) DH 5
COMMON /TFN1/A(169,170) /TFN2/AGR(169),SS(169),CC(169) DH 10
1/DIAG/DPR,AG(169) DH 11
TYPE DOUBLE DPR,AG, AGR,SS,CC DH 15
DIMENSION JJ(3),II(3),KK(3),ANGLE(3) DH 20
IF(MODE)1,2,1 DH 25
1 IF(IR)4,4,3 DH 30
4 CONTINUE DH 35
RETURN DH 40
3 PRINT 5, (JJ(L),II(L),KK(L),ANGLE(L),L=1,IR) DH 45

```

Table A1 (Continued)
Program Listing

IR = 0	DH 50
GO TO 4	DH 55
2 IR = IR+1	DH 60
COSU = (CC(I)-CC(J)*CC(K))/(SS(J)*SS(K))	DH 65
ANGLE(IR) = DPR*ACOSF(COSU)	DH 70
JJ(IR) = J \$ II(IR) = I \$ KK(IR) = K	DH 75
IF(IR-3)4,3,3	DH 80
5 FORMAT(3(4X,3I4,F16.5,7X))	DH 85
END	DH 90
SUBROUTINE GAUSS2(N,M,EP,A,X,KER)	001
DIMENSIION A(169,170),X(169,1)	002
NPM=N+M	003
10 DO 34 L=1,N	004
KP=0	005
Z=0.0	006
DO 12 K=L,N	007
IF(Z-ABSF(A(K,L)))11,12,12	008
11 Z=ABSF(A(K,L))	009
KP=K	01
12 CONTINUE	011
IF(L-KP)13,20,20	012
13 DO 14 J=L,NPM	013
Z=A(L,J)	014
A(L,J)=A(KP,J)	015
14 A(KP,J)=Z	016
20 IF(ABSF(A(L,L))-EP)50,50,30	017
30 IF(L-N)31,40,40	018
31 LP1=L+1	019
DO 34 K=LP1,N	020
IF(A(K,L))32,34,32	021
32 RATIO=A(K,L)/A(L,L)	022
DO 33 J=LP1,NPM	023
33 A(K,J)=A(K,J)-RATIO*A(L,J)	024
34 CONTINUE	025
40 DO 43 I=1,N	026
II=N+1-I	027
DO 43 J=1,M	028
JPN=J+N	029
S=0.0	03
IF(II-N)41,43,43	031
41 IIPI=II+1	032
DO 42 K=IIPI,N	033
42 S=S+A(II,K)*X(K,J)	034
43 X(II,J)=(A(II,JPN)-S)/A(II,II)	035
KER=1	036
GO TO 75	037
50 KER=2	038
75 CONTINUE	039
END	040

Appendix B
COMPUTER PRINTOUT OF PH POLYHEDRA PARAMETERS



POLYHEDRON WITH 42 FACES

Double precision

NRL REPORT 6706

INITIAL GUESS

$$\begin{aligned} AG(1) \text{ THRU } AG(-1) &= 1.000001 \\ AG(-2) &= -2.000001 \\ AG(-3) &= -1.200001 \end{aligned}$$

FACE ANGLES -- EXPRESSED AS EXCESS OVER 120 DEG

4 ITERATIONS REQUIRED

INDEX	EXCESS (DEG)	INDEX	EXCESS (DEG)	INDEX	EXCESS (DEG)
1	1.71747441146+000	2	-3.43494882292+000	3	-1.20000000000+001

HEXAGON DIAGONALS

SHORT DIAGONALS

$$\begin{aligned} L(-1, 1) &= 1.7013016167+000 \\ L(-2, 1) &= 1.7468435031+000 \\ L(-1, 2) &= 1.7468435031+000 \end{aligned}$$

$$\begin{aligned} L(1, 1) &= 1.7013016167+000 \\ L(2, 1) &= 1.7468435031+000 \\ L(2, 2) &= 1.7468435031+000 \end{aligned}$$

$$\begin{aligned} L(-1, 1) &= 1.9734303107+000 \\ L(-2, 2) &= 2.0514622242+000 \\ L(-1, 1) &= 1.9734303107+000 \end{aligned}$$

DIHEDRAL ANGLES -- EXPRESSED IN DEG

ORDERED TRIPLET	DIHD ANGLE	ORDERED TRIPLET	DIHD ANGLE	ORDERED TRIPLET	DIHD ANGLE
3 1 1	148.28225256	1 3 1	144.0000000		Execution time--1 sec

POLYHEDRON WITH 92 FACES

INITIAL GUESS

AG(1) THRU AG(2) =	1.000+001
AG(3) =	-2.00-001
AG(4) =	-1.00-001
AG(5) =	-1.20+001

FACE ANGLES -- EXPRESSED AS EXCESS OVER 120 DEG

5 ITERATIONS REQUIRED

INDEX	EXCESS (DEG)	INDEX	EXCESS (DEG)	INDEX	EXCESS (DEG)
1	4.89409909706+000 1.55224878141+001	3	-9.78819819411+000 4	-1.55224878141+001	-1.70n00000000+001
2					

HEXAGON DIAGONALS

SHORT DIAGONALS

L(1, 1) = 1.6404215939+000	L(1, 1) = 1.6404215939+000
L(3, 1) = 1.7731674500+000	L(3, 1) = 1.7731674500+000
L(1, 3) = 1.7731674500+000	L(3, 1) = 1.7731674500+000
L(4, 4) = 1.8512295868+000	L(2, 2) = 1.5811388301+000
L(2, 2) = 1.5811388301+000	L(4, 4) = 1.8512295868+000
L(4, 4) = 1.8512295868+000	L(2, 2) = 1.5811388301+000

LONG DIAGONALS

L(1, 1) = 1.9211931203+000	L(1, 1) = 1.9211931203+000
L(3, 3) = 2.1441228056+000	L(3, 3) = 2.1441228056+000
L(1, 1) = 1.9211931203+000	L(1, 1) = 1.9211931203+000
L(2, 4) = 1.9816788295+000	L(2, 4) = 1.9816788295+000
L(2, 4) = 1.9816788295+000	L(2, 4) = 1.9816788295+000
L(4, 2) = 1.9816788295+000	L(4, 2) = 1.9816788295+000

DIHEDRAL ANGLES -- EXPRESSED IN DEG

ORDERED TRIPLET	DIHD ANGLE	ORDERED TRIPLET	DIHD ANGLE	ORDERED TRIPLET	DIHD ANGLE
5 1 1	163.7329627	1 5 1	161.0465242	1 4 1	149.0991258
4 1 1	154.2123482				

Double precision

POLYHEDRON WITH 162 FACES

INITIAL GUESS

```

AG(1) THRU AG( 6 ) = 1.00-001
AG( 7 ) = -2.00-001
AG( 8 ) = -2.00-001
AG( 9 ) = -1.20+001

```

FACE ANGLES -- EXPRESSED AS EXCESS OVER 120 DEG

5 ITERATIONS REQUIRED

INDEX	EXCESS (DEG)	INDEX	EXCESS (DEG)	INDEX	EXCESS (DEG)
1	5.57314296850+000	4	-1.48768290060+001	7	-1.11462859370+001
2	6.56478188832+000	5	5.19027378783+000	8	-1.31295637766+001
3	2.05345842607+001	6	-1.16147382428+000	9	-1.20000000000+001

HEXAGON DIAGONALS

SHORT DIAGONALS

L(1,	1)	=	1.6267470763+000
L(7,	1)	=	1.7786184360+000
L(1,	7)	=	1.7786184360+000
L(2,	2)	=	1.6063676125+000
L(8,	2)	=	1.7864665085+000
L(2,	8)	=	1.7864665085+000
L(4,	4)	=	1.8825559144+000
L(3,	5)	=	1.5880144422+000
L(4,	6)	=	1.775526429+000

LONG DIAGONALS

L(1,	1)	=	1.6267470763+000
L(7,	1)	=	1.7786184360+000
L(1,	7)	=	1.7786184360+000
L(2,	2)	=	1.6063676125+000
L(8,	2)	=	1.7864665085+000
L(2,	8)	=	1.7864665085+000
L(5,	5)	=	1.7218262405+000
L(6,	4)	=	1.775526429+000
L(3,	5)	=	1.5880144422+000

DIHEDRAL ANGLES -- EXPRESSED IN DEG

ORDERED TRIPLET	DIHD ANGLE	ORDERED TRIPLET	DIHD ANGLE	ORDERED TRIPLET	DIHD ANGLE
9 1 1	169.8719289	1 9 1	168.1344552	1 4 2	158.4105967
4 2 1	162.1734845	4 1 2	161.9395794	6 6 6	158.7140604

POLYHEDRON WITH 252 FACES

INITIAL GUESS

AG(1) THRU AG(10) = 1.00-001
 AG(11) = -2.00-001
 AG(12) = -2.00-001
 AG(13) = -1.20+001

FACE ANGLES -- EXPRESSED AS EXCESS & VFR 120 DEG

5 ITERATIONS REQUIRED

INDEX	EXCESS(DEG)	INDEX	EXCESS(DEG)	INDEX	EXCESS(DEG)
1	5.79938335413+000	6	6.16161441894+000	11	*1.15987667083+001
2	7.11971008402+000	7	*1.62259867661+001	12	*1.42394201680+001
3	2.23353435247+001	8	1.25090061063+001	13	*1.20000000000+001
4	-1.43102793200+001	9	-4.10159633326+000		
5	6.18003482157-002	10	-5.88832779983-001		

HEXAGON DIAGONALS

SHORT DIAGONALS

LONG DIAGONALS

L(1, 1) = 1.62214022991+000	L(1, 1) = 1.62214022991+000	L(1, 1) = 1.9056072321+000
L(11, 1) = 1.7804207064+000	L(11, 1) = 1.7804207064+000	L(11, 1) = 2.1698978918+000
L(1, 11) = 1.7804207064+000	L(11, 1) = 1.7804207064+000	L(1, 11) = 1.9056072321+000
L(2, 2) = 1.5947527491+000	L(2, 2) = 1.5947527491+000	L(2, 2) = 1.8823486209+000
L(12, 2) = 1.7908000031+000	L(12, 2) = 1.7908000031+000	L(12, 2) = 2.069646512+000
L(2, 12) = 1.7908000031+000	L(12, 2) = 1.7908000031+000	L(2, 12) = 1.8823486209+000
L(4, 4) = 1.8929345522+000	L(5, 5) = 1.7832918484+000	L(4, 4) = 2.0918065772+000
L(3, 5) = 1.5940061023+000	L(6, 4) = 1.7325898654+000	L(3, 6) = 1.7740293707+000
L(4, 6) = 1.7325898654+000	L(3, 5) = 1.5940061023+000	L(4, 5) = 2.0918065772+000
L(8, 8) = 1.5735901440+000	L(9, 9) = 1.7268894278+000	L(9, 8) = 1.9280602126+000
L(7, 9) = 1.8306862588+000	L(10, 8) = 1.6951558780+000	L(7, 10) = 2.1187158885+000
L(8, 10) = 1.6951558780+000	L(7, 9) = 1.8306862588+000	L(8, 9) = 1.9280602126+000

DIHEDRAL ANGLES -- EXPRESSED IN DEG

ORDERED TRIPLET	DIHD ANGLE	ORDERED TRIPLET	DIHD ANGLE	ORDERED TRIPLET	DIHD ANGLE
13 1 1	173.0515653	1 13 1	171.8447502	1 4 2	164.5754792
4 2 1	167.2742324	4 1 2	167.0519225	2 7 2	161.3109621
7 2 2	164.7476975	6 9 9	162.9905576	9 6 9	164.7790287

POLYHEDRON WITH 362 FACES

INITIAL GUESS

AG(1) THRU AG(14) = 1.00-001
 AG(15) = -2.00-001
 AG(16) = -2.00-001
 AG(17) = -2.00-001
 AG(18) = -1.00-001
 AG(19) = -1.20+001

FACE ANGLES -- EXPRESSED AS EXCESS OVER 120 DEG

6 ITERATIONS REQUIRED

INDEX	EXCESS (DEG)	INDEX	EXCESS (DEG)	INDEX	EXCESS (DEG)
1	5.89243490437+000	8	9.19406699004+000	15	-1.17848698087+001
2	7.47960643402+000	9	1.59504648059+001	16	-1.49592128680+001
3	7.30418785726+000	10	"5.21452940058+000	17	-1.46083757145+001
4	8.32919659603+000	11	-4.76998790453+000	18	-8.32919659603+000
5	2.30989660157+001	12	3.44499497273+000	19	-1.20000000000+001
6	-1.41449673217+001	13	6.66442526523+000		
7	-2.00154918113+000	14	-1.60753677388+001		

HEXAGON DIAGONALS

SHORT DIAGONALS

LONG DIAGONALS

L(1, 1) = 1.6202381111+000 L(1, 1) = 1.6202381111+000 L(1, 1) = 1.9039883237+000
L(15, 1) = 1.7811599575+000 L(15, 1) = 1.7811599575+000 L(15, 1) = 2.1725307942+000
L(1, 15) = 1.7811599575+000 L(15, 1) = 1.7811599575+000 L(1, 1) = 1.9039883237+000
L(2, 2) = 1.5871399389+000 L(2, 2) = 1.5871399389+000 L(2, 2) = 1.8759032986+000
L(16, 2) = 1.7935880287+000 L(16, 2) = 1.7935880287+000 L(16, 2) = 2.2169580167+000
L(2, 16) = 1.7935880287+000 L(16, 2) = 1.7935880287+000 L(2, 2) = 1.8759032986+000
L(3, 3) = 1.5908583718+000 L(3, 3) = 1.5908583718+000 L(3, 3) = 1.8790503876+000
L(17, 3) = 1.7922313139+000 L(17, 3) = 1.7922313139+000 L(17, 3) = 2.2120930826+000
L(3, 17) = 1.7922313139+000 L(17, 3) = 1.7922313139+000 L(3, 3) = 1.8790503876+000
L(18, 18) = 1.8000993612+000 L(4, 4) = 1.6548554287+000 L(4, 18) = 1.9947190780+000
L(4, 4) = 1.6548554287+000 L(18, 18) = 1.8000993612+000 L(4, 4) = 1.6548554287+000 L(4, 18) = 1.9947190780+000
L(18, 18) = 1.8000993612+000 L(4, 4) = 1.6548554287+000 L(18, 4) = 1.6548554287+000 L(18, 4) = 1.9947190780+000
L(6, 6) = 1.8971946779+000 L(7, 7) = 1.8066261668+000 L(7, 7) = 2.1041676617+000
L(5, 7) = 1.5957470558+000 L(8, 6) = 1.7143206755+000 L(5, 8) = 1.7444415718+000
L(6, 8) = 1.7143206755+000 L(5, 7) = 1.5957470558+000 L(6, 7) = 2.1041676617+000
L(10, 14) = 1.8540436689+000 L(11, 13) = 1.7613268777+000 L(11, 14) = 2.1485057358+000
L(9, 11) = 1.6847681558+000 L(12, 14) = 1.7872477234+000 L(9, 12) = 1.8454779745+000
L(10, 12) = 1.6889364645+000 L(9, 13) = 1.5752113159+000 L(10, 13) = 1.9786346760+000

DIHEDRAL ANGLES -- EXPRESSED IN DEG

ORDERED TRIPLET	DIHD ANGLE	ORDERED TRIPLET	DIHD ANGLE	ORDERED TRIPLET	DIHD ANGLE
19 1 1	174.9105495	1 19 1	174.0221461	1 6 2	168.4796761
6 2 1	170.5169269	6 1 2	170.3172828	2 14 3	164.8922999
14 3 2	167.6670095	14 2 3	167.6962884	8 10 10	166.4898206
10 8 10	168.4965585	12 18 12	165.1596603	18 12 12	166.7047029

Execution time--2 sec

POLYHEDRON WITH 492 FACES

INITIAL GUESS

AG(1) THRU AG(21) =	1.00-001
AG(22) =	-2.00-001
AG(23) =	-2.00-001
AG(24) =	-2.00-001
AG(25) =	-1.20+001

FACE ANGLES -- EXPRESSED AS EXCESS OVER 120 DEG

6 ITERATIONS REQUIRED

INDEX	EXCESS(DEG)	INDEX	EXCESS(DEG)	INDEX	EXCESS(DEG)
1	5.93670840891+000	10	1.26326309437+000	19	7.64530823283+000
2	7.70192493691+000	11	-7.52141262264+000	20	3.60229851992+000
3	7.51689393055+000	12	1.35737404957+001	21	-1.60576810065+001
4	2.34673776506+001	13	-9.91430383357+000	22	-1.18734168178+001
5	-1.41017460378+001	14	3.30312613353+000	23	-1.54038498738+001
6	-2.90222318633+000	15	-3.51385095565-001	24	-1.50337878611+001
7	1.05405607977+001	16	1.76736255879+001	25	-1.20000000000+001
8	-1.60408320359+001	17	-5.65887690371+000		
9	1.05178392349+001	18	-7.20467443046+000		

HEXAGON DIAGONALS

SHORT DIAGONALS

LONG DIAGONALS

L(1, 1) = 1.6193315913+000 L(1, 1) = 1.6193315913+000 L(1, 1) = 1.9032169615+000
L(22, 1) = 1.7815112775+000 L(22, 1) = 1.7815112775+000 L(22, 1) = 2.1737824319+000
L(1, 22) = 1.7815112775+000 L(22, 1) = 1.7815112775+000 L(1, 1) = 1.9032169615+000
L(2, 2) = 1.5824059747+000 L(2, 2) = 1.5824059747+000 L(2, 2) = 1.8718997486+000
L(23, 2) = 1.7953014355+000 L(23, 2) = 1.7953014355+000 L(23, 2) = 2.2231072443+000
L(2, 23) = 1.7953014355+000 L(23, 2) = 1.7953014355+000 L(2, 2) = 1.8718997486+000
L(3, 3) = 1.5863476193+000 L(3, 3) = 1.5863476193+000 L(3, 3) = 1.8752329906+000
L(24, 3) = 1.7938758745+000 L(24, 3) = 1.7938758745+000 L(24, 3) = 2.2179906530+000
L(3, 24) = 1.7938758745+000 L(24, 3) = 1.7938758745+000 L(3, 3) = 1.8752329906+000
L(5, 5) = 1.8992198702+000 L(6, 6) = 1.8165826112+000 L(6, 5) = 2.1095235934+000
L(4, 6) = 1.5962016855+000 L(7, 5) = 1.7061713659+000 L(4, 7) = 1.730918436+000
L(5, 7) = 1.7061713659+000 L(4, 6) = 1.5962016855+000 L(5, 6) = 2.1095235934+000
L(9, 9) = 1.5755826539+000 L(10, 10) = 1.6627315674+000 L(10, 9) = 1.9025695824+000
L(8, 10) = 1.8164166768+000 L(11, 9) = 1.7429695589+000 L(8, 11) = 2.1707179999+000
L(9, 11) = 1.7429695589+000 L(8, 10) = 1.8164166768+000 L(9, 10) = 1.9025695824+000
L(13, 13) = 1.8380900808+000 L(14, 14) = 1.7289762558+000 L(14, 13) = 2.0440191060+000
L(12, 14) = 1.6391615173+000 L(15, 13) = 1.7601525047+000 L(12, 15) = 1.8953164255+000
L(13, 15) = 1.7601525047+000 L(12, 14) = 1.6391615173+000 L(13, 14) = 2.0440191060+000
L(17, 21) = 1.8651119083+000 L(18, 20) = 1.7948657321+000 L(18, 21) = 2.1645865391+000
L(16, 18) = 1.6805763280+000 L(19, 21) = 1.7626258610+000 L(16, 19) = 1.7996411021+000
L(17, 19) = 1.6657973320+000 L(16, 20) = 1.5754015067+000 L(17, 20) = 2.0024418951+000

DIHEDRAL ANGLES -- EXPRESSED IN DEG

ORDERED TRIPLET	DIHD ANGLE	ORDERED TRIPLET	DIHD ANGLE	ORDERED TRIPLET	DIHD ANGLE
25 1 1	176.0954492	1 25 1	175.4122517	1 5 2	171.0702152
5 2 1	172.6634026	5 1 2	172.4912269	2 21 3	167.7955417
21 3 2	170.0512300	21 2 3	170.0762005	3 8 3	166.6426390
8 3 3	169.1461694	7 17 17	169.1751585	17 7 17	170.9876047
19 13 10	167.3491037	13 10 19	168.5021426	13 19 10	169.3596392
15 15 15	168.2208859				

Execution time--3 sec

INITIAL GUESS

```

AG(1) THRU AG( 28) = 1.00-001
AG( 29) = -2.00-001
AG( 30) = -2.00-001
AG( 31) = -2.00-001
AG( 32) = -2.00-001
AG( 33) = -1.20+001

```

FACE ANGLES -- EXPRESSED AS EXCESS OVER 120 DEG

6 ITERATIONS REQUIRED

INDEX	EXCESS(DEG)	INDEX	EXCESS(DEG)	INDEX	EXCESS(DEG)
1	5.96009074299+000	12	5.19817831138+000	23	1.27913516534+001
2	7.83654734975+000	13	-1.16581900537+001	24	2.48259533989-002
3	7.74998382861+000	14	8.93814917103+000	25	-9.38902519219+000
4	7.51387069857+000	15	-2.99213350587+000	26	5.43626635817+000
5	2.36633195539+001	16	-2.33841276634-001	27	7.14463946069+000
6	-1.40915833801+001	17	4.86045141964+001	28	-1.6008058235+001
7	-3.34395330827+000	18	-5.85856927837+000	29	-1.19201814860+001
8	1.12077538228+001	19	-8.5898594624+000	30	-1.56730946995+001
9	1.65375018962+001	20	9.8941975316+000	31	-1.54999676572+001
10	-1.06255729538+001	21	2.06454406947+000	32	-1.50277413971+001
11	-2.42267149950-001	22	-1.61457006144+001	33	-1.200000000000+001

HEXAGON DIAGONALS

L(1, 1) = 1.6188524374+000	L(1, 1) = 1.6188524374+000	L(1, 1) = 1.6188524374+000	L(1, 1) = 1.9028092952+000
L(29, 1) = 1.7816967142+000	L(29, 1) = 1.7816967142+000	L(29, 1) = 1.7816967142+000	L(29, 1) = 2.1744431813+000
L(1, 29) = 1.7816967142+000	L(29, 1) = 1.7816967142+000	L(1, 1) = 1.7816967142+000	L(1, 1) = 1.9028092952+000
L(2, 2) = 1.5795277913+000	L(2, 2) = 1.5795277913+000	L(2, 2) = 1.5795277913+000	L(2, 2) = 1.8694673155+000
L(30, 2) = 1.7963356849+000	L(30, 2) = 1.7963356849+000	L(30, 2) = 1.7963356849+000	L(30, 2) = 2.2268218927+000
L(2, 30) = 1.7963356849+000	L(30, 2) = 1.7963356849+000	L(2, 2) = 1.8694673155+000	L(2, 2) = 1.8694673155+000

LONG DIAGONALS

L(3, 3) = 1.5813794931+000	L(3, 3) = 1.5813794931+000	L(3, 3) = 1.5813794931+000
L(31, 3) = 1.7956709369+000	L(31, 3) = 1.7956709369+000	L(31, 3) = 1.7956709369+000
L(3, 31) = 1.7956709369+000	L(31, 3) = 1.7956709369+000	L(3, 31) = 1.7956709369+000
L(4, 4) = 1.58644118848+000	L(4, 4) = 1.58644118848+000	L(4, 4) = 1.58644118848+000
L(32, 4) = 1.7938525433+000	L(32, 4) = 1.7938525433+000	L(32, 4) = 1.7938525433+000
L(4, 32) = 1.7938525433+000	L(32, 4) = 1.7938525433+000	L(4, 32) = 1.7938525433+000
L(6, 6) = 1.9002889851+000	L(7, 7) = 1.8214232228+000	L(7, 6) = 2.1121625145+000
L(5, 7) = 1.596308550+000	L(8, 6) = 1.7021360313+000	L(5, 8) = 1.7240749764+000
L(6, 8) = 1.7021360313+000	L(5, 7) = 1.5963085500+000	L(6, 7) = 2.1121625145+000
L(10, 10) = 1.8578615478+000	L(11, 11) = 1.7756161382+000	L(11, 10) = 2.0733665732+000
L(9, 11) = 1.6320172230+000	L(12, 10) = 1.7299327585+000	L(9, 12) = 1.8296214524+000
L(10, 12) = 1.799327585+000	L(9, 11) = 1.6320172230+000	L(10, 11) = 2.0733665732+000
L(14, 14) = 1.6215333411+000	L(15, 15) = 1.7300065525+000	L(15, 14) = 1.9507084112+000
L(13, 15) = 1.8047055688+000	L(16, 14) = 1.7053520618+000	L(13, 16) = 2.0856637856+000
L(14, 16) = 1.7053520618+000	L(13, 15) = 1.8047055688+000	L(14, 15) = 1.9507084112+000
L(18, 22) = 1.8709159096+000	L(19, 21) = 1.8118343944+000	L(19, 22) = 2.1737269522+000
L(17, 19) = 1.6786842623+000	L(20, 22) = 1.7497852780+000	L(17, 20) = 1.7746274964+000
L(18, 20) = 1.6525998722+000	L(17, 21) = 1.5744546340+000	L(18, 21) = 2.0135693463+000
L(24, 28) = 1.8326650244+000	L(25, 27) = 1.7775246865+000	L(25, 28) = 2.1836975202+000
L(23, 25) = 1.7322674142+000	L(26, 28) = 1.7909936944+000	L(23, 26) = 1.8537252140+000
L(24, 26) = 1.6443971180+000	L(23, 27) = 1.5759349150+000	L(24, 27) = 1.9356181369+000

DIHEDRAL ANGLES -- EXPRESSED IN DEG

ORDERED TRIPLET	DIHD ANGLE	ORDERED TRIPLET	DIHD ANGLE	ORDERED TRIPLET	DIHD ANGLE	ORDERED TRIPLET	DIHD ANGLE
33	1	1	176.8992401	1	33	1	176.3560030
6	2	1	174.1475149	6	1	2	174.0012806
22	3	2	171.8800846	22	2	3	171.8896565
28	4	3	170.6233924	28	3	4	170.6534038
18	8	18	172.7474427	20	10	24	169.2731396
10	20	24	171.2934020	26	13	26	168.6485794
12	15	15	169.5046329	15	12	15	170.3824185

Execution time--3 sec

INITIAL GUESS

$AG(1)$ THRU $AG(35) = 1.00-001$
 $AG(-36) = -2.00-001$
 $AG(-37) = -2.00-001$
 $AG(-38) = -2.00-001$
 $AG(-39) = -2.00-001$
 $AG(-40) = -1.00-001$
 $AG(-41) = -1.20+001$

FACE ANGLES -- EXPRESSED AS EXCESS OVER 120 DEG

6 ITERATIONS REQUIRED

INDEX	EXCESS (DEG)	INDEX	EXCESS (DEG)	INDEX	EXCESS (DEG)	INDEX	EXCESS (DEG)
1	5.97345441161+000	15	9.48160989880+000	29	-1.61150769344+001		
2	7.91944017005+000	16	3.77501885116+000	30	-1.25870913907+001		
3	7.94491300635+000	17	-1.06547840126+001	31	-4.6181342199+000		
4	7.62508362983+000	18	1.91428248797+001	32	-3.10968634382+000		
5	5.66610183700+000	19	-5.95954204753+000	33	2.52248215705+000		
6	2.37757395567+001	20	-9.33318857592+000	34	5.04484634531+000		
7	-1.40902234423+001	21	1.11407498275+001	35	-1.24265988272+001		
8	-3.58380733479+000	22	1.25800605677+000	36	-1.19469088232+001		
9	1.15723219975+001	23	-1.62488501406+001	37	-1.58388803401+001		
10	1.82568539192+001	24	1.41735216004+001	38	-1.58898260127+001		
11	-1.10271541740+001	25	-6.16893751354-001	39	-1.52501672597+001		
12	-2.42392182307+000	26	-1.06859747525+001	40	-5.66610183700+000		
13	8.64529807493+000	27	8.04851253997+000	41	-1.20000000000+001		
14	-1.58584734873+001	28	5.19591129794+000				

HEXAGON DIAGONALS

SHORT DIAGONALS

$L(-1, 1) = 1.61857846662+000$	$L(-1, 1) = 1.61857846662+000$	$L(-1, 1) = 1.9025762143+000$
$L(-36, 1) = 1.7818026632+000$	$L(-36, 1) = 1.7818026632+000$	$L(-36, 1) = 2.1748207305+000$
$L(-1, 36) = 1.7818026632+000$	$L(-36, 1) = 1.7818026632+000$	$L(-1, 1) = 1.9025762143+000$
$L(-2, 2) = 1.5777512311+000$	$L(-2, 2) = 1.5777512311+000$	$L(-2, 2) = 1.8679665273+000$
$L(-37, 2) = 1.7969712836+000$	$L(-37, 2) = 1.7969712836+000$	$L(-37, 2) = 2.2291057940+000$
$L(-2, 37) = 1.7969712836+000$	$L(-37, 2) = 1.7969712836+000$	$L(-2, 2) = 1.8679665273+000$
$L(-3, 3) = 1.57772046333+000$	$L(-3, 3) = 1.57772046333+000$	$L(-3, 3) = 1.8675048743+000$
$L(-38, 3) = 1.7971664132+000$	$L(-38, 3) = 1.7971664132+000$	$L(-38, 3) = 2.2298071168+000$
$L(-3, 38) = 1.7971664132+000$	$L(-38, 3) = 1.7971664132+000$	$L(-3, 3) = 1.8675048743+000$

LONG DIAGONALS

ORDERED TRIPLET	DIHEDRAL ANGLE	ORDERED TRIPLET	DIHEDRAL ANGLE	ORDERED TRIPLET	DIHEDRAL ANGLE
41	1	177.4710135	1	41	1
7	2	175.2143630	7	1	2
23	3	173.2711383	23	2	3
29	4	171.9695374	29	3	3
14	4	171.5312209	9	19	4
21	11	170.8806589	11	25	19
27	35	169.9633644	35	16	19
13	31	170.6895941	31	13	21
40	33	171.1682467	33	33	25
DIHEDRAL ANGLES -- EXPRESSED IN DEG					
41	1	1.5840449016+000	L(4, 4) = 1.5840449016+000	L(4, 4) = 1.5840449016+000	L(4, 4) = 1.8732854161+000
7	2	1.7947099841+000	L(39, 4) = 1.7947099841+000	L(39, 4) = 1.7947099841+000	L(39, 4) = 2.2209839269+000
23	3	1.6805079614+000	L(5, 5) = 1.6805079614+000	L(5, 5) = 1.6805079614+000	L(5, 5) = 1.8732854161+000
29	4	1.7793598021+000	L(40, 40) = 1.7793598021+000	L(40, 40) = 1.7793598021+000	L(40, 40) = 1.997555846+000
14	4	1.7793598021+000	L(5, 5) = 1.6805079614+000	L(40, 40) = 1.6805079614+000	L(40, 40) = 1.997555846+000
21	11	1.9008998726+000	L(8, 8) = 1.8240421567+000	L(8, 8) = 1.8240421567+000	L(8, 8) = 2.1136039135+000
27	35	1.5963228493+000	L(9, 7) = 1.6999342948+000	L(9, 7) = 1.6999342948+000	L(9, 7) = 1.7202823290+000
13	31	1.6999342948+000	L(6, 8) = 1.5963228493+000	L(6, 8) = 1.5963228493+000	L(6, 8) = 2.1136039135+000
10	12	1.8687627055+000	L(12, 12) = 1.8024967529+000	L(12, 12) = 1.8024967529+000	L(12, 12) = 2.0900810292+000
11	13	1.6279557786+000	L(13, 11) = 1.7105121992+000	L(13, 11) = 1.7105121992+000	L(13, 11) = 1.7890282624+000
14	16	1.5775410525+000	L(10, 12) = 1.6279557786+000	L(10, 12) = 1.6279557786+000	L(10, 12) = 2.0900810292+000
15	17	1.8067733528+000	L(16, 16) = 1.6317224685+000	L(16, 16) = 1.6317224685+000	L(16, 16) = 1.8905314545+000
17	19	1.7640483274+000	L(17, 15) = 1.7640483274+000	L(17, 15) = 1.7640483274+000	L(17, 17) = 2.1925684893+000
20	22	1.8742159077+000	L(14, 16) = 1.8087733528+000	L(14, 16) = 1.8087733528+000	L(14, 16) = 1.8905314545+000
21	23	1.6777256143+000	L(20, 22) = 1.8209398825+000	L(20, 22) = 1.8209398825+000	L(20, 23) = 2.1791801905+000
24	26	1.6449516276+000	L(21, 23) = 1.7429243883+000	L(21, 23) = 1.7429243883+000	L(21, 21) = 1.7604569417+000
25	27	1.6314076209+000	L(18, 22) = 1.5733438187+000	L(18, 22) = 1.5733438187+000	L(19, 22) = 2.0191134194+000
28	29	1.8421909341+000	L(26, 28) = 1.7979591019+000	L(26, 28) = 1.7979591019+000	L(26, 29) = 2.1929035585+000
31	32	1.7266423217+000	L(27, 29) = 1.7755979287+000	L(27, 29) = 1.7755979287+000	L(24, 27) = 1.8228291128+000
33	34	1.6314076209+000	L(24, 28) = 1.5747841743+000	L(24, 28) = 1.5747841743+000	L(25, 28) = 1.9533317081+000
35	36	1.8312346171+000	L(32, 34) = 1.7536422147+000	L(32, 34) = 1.7536422147+000	L(32, 35) = 2.1119897793+000
32	33	1.6903545171+000	L(33, 35) = 1.7743829487+000	L(33, 35) = 1.7743829487+000	L(33, 33) = 1.880968842+000
33	34	1.7042792828+000	L(30, 34) = 1.61364633997+000	L(30, 34) = 1.61364633997+000	L(31, 34) = 1.9905101185+000

Execution time--4 sec

Double precision

POLYHEDRON WITH 1002 FACES

INITIAL GUESS

```

AG(1) THRU AG( 45) = 1.00=001
AG( 46) = -2.00-001
AG( 47) = -2.00-001
AG( 48) = -2.00-001
AG( 49) = -2.00-001
AG( 50) = -2.00-001
AG( 51) = -1.20+001

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FACE ANGLES -- EXPRESSED AS EXCESS OVER 120 DEG

6 ITERATIONS REQUIRED

INDEX	EXCESS (DEG)	INDEX	EXCESS (DEG)	INDEX	EXCESS (DEG)
1	5.98157834646+000	18	9.93786238121+000	35	2.73912579420+000
2	7.97202973105+000	19	-7.30626816760+000	36	-1.16225172578+001
3	8.09125321546+000	20	2.42086370270+000	37	6.33590807921+000
4	7.78677368561+000	21	-1.67053451411-001	38	7.35759847759+000
5	7.59718579817+000	22	1.94723307848+001	39	-1.58658489387+001
6	2.384423567876+001	23	-6.01585340808+000	40	1.49177993343+001
7	-1.40910395531+001	24	-9.79784746559+000	41	-5.598860193834+000
8	-3.72485534190+000	25	1.18715040194+001	42	-5.18830646907+000
9	1.17875530024+001	26	8.05593128104-001	43	6.40268838024+000
10	1.93008236163+001	27	-1.63357270587+001	44	2.34692761652+000
11	-1.12854773196+001	28	1.50450491639+001	45	-1.28805069236+001
12	-3.74000726715+000	29	-9.51161407004-001	46	-1.19631566929+001
13	1.07501455571+001	30	-1.15725530035+001	47	-1.59440594621+001
14	-1.31690279330+001	31	9.68017623559+000	48	1.61825064309+001
15	8.82127760627+000	32	4.07882514544+000	49	-1.55735473712+001
16	4.10511201861-001	33	-1.62803561346+001	50	-1.51943715963+001
17	-5.29454968322+000	34	1.10557338455+001	51	-1.20000000000+001

HEXAGON DIAGONALS

SHORT DIAGONALS

LONG DIAGONALS

$L(1, 1) = 1.6184118728*000$	$L(1, 1) = 1.6184118728*000$	$L(1, 1) = 1.6184118728*000$	$L(1, 1) = 1.9024344903*000$
$L(46, 1) = 1.7818670590*000$	$L(46, 1) = 1.7818670590*000$	$L(46, 1) = 1.7818670590*000$	$L(46, 1) = 2.1750502160*000$
$L(1, 46) = 1.7818670590*000$	$L(1, 46) = 1.7818670590*000$	$L(1, 46) = 1.7818670590*000$	$L(1, 1) = 1.9024344903*000$
$L(2, 2) = 1.5766224184*000$	$L(2, 2) = 1.5766224184*000$	$L(2, 2) = 1.5766224184*000$	$L(2, 2) = 1.8670131896*000$
$L(47, 2) = 1.7973740380*000$	$L(47, 2) = 1.7973740380*000$	$L(47, 2) = 1.7973740380*000$	$L(47, 2) = 2.2305534324*000$
$L(2, 47) = 1.7973740380*000$	$L(2, 47) = 1.7973740380*000$	$L(2, 47) = 1.7973740380*000$	$L(2, 2) = 1.8670131896*000$
$L(3, 3) = 1.5740584192*000$	$L(3, 3) = 1.5740584192*000$	$L(3, 3) = 1.5740584192*000$	$L(3, 3) = 1.8648484944*000$
$L(48, 3) = 1.7982857027*000$	$L(48, 3) = 1.7982857027*000$	$L(48, 3) = 1.7982857027*000$	$L(48, 3) = 2.2338314686*000$
$L(3, 48) = 1.7982857027*000$	$L(3, 48) = 1.7982857027*000$	$L(3, 48) = 1.7982857027*000$	$L(3, 3) = 1.8648484944*000$
$L(4, 4) = 1.5805929529*000$	$L(4, 4) = 1.5805929529*000$	$L(4, 4) = 1.5805929529*000$	$L(4, 4) = 1.8703673657*000$
$L(49, 4) = 1.7959535828*000$	$L(49, 4) = 1.7959535828*000$	$L(49, 4) = 1.7959535828*000$	$L(49, 4) = 2.2254492716*000$
$L(4, 49) = 1.7959535828*000$	$L(4, 49) = 1.7959535828*000$	$L(4, 49) = 1.7959535828*000$	$L(4, 4) = 1.8703673657*000$
$L(5, 5) = 1.5846392219*000$	$L(5, 5) = 1.5846392219*000$	$L(5, 5) = 1.5846392219*000$	$L(5, 5) = 1.8737879986*000$
$L(50, 5) = 1.7944950533*000$	$L(50, 5) = 1.7944950533*000$	$L(50, 5) = 1.7944950533*000$	$L(50, 5) = 2.2202124964*000$
$L(5, 50) = 1.7944950533*000$	$L(5, 50) = 1.7944950533*000$	$L(5, 50) = 1.7944950533*000$	$L(5, 5) = 1.8737879986*000$
$L(7, 7) = 1.9012711879*000$	$L(7, 7) = 1.9012711879*000$	$L(7, 7) = 1.9012711879*000$	$L(7, 7) = 2.1144554774*000$
$L(6, 8) = 1.5963142682*000$	$L(6, 8) = 1.5963142682*000$	$L(6, 8) = 1.5963142682*000$	$L(6, 9) = 1.7180226731*000$
$L(7, 9) = 1.6986360685*000$	$L(7, 9) = 1.6986360685*000$	$L(7, 9) = 1.6986360685*000$	$L(7, 8) = 2.1144554774*000$
$L(11, 11) = 1.8751766370*000$	$L(11, 12) = 1.8751766370*000$	$L(11, 12) = 1.8751766370*000$	$L(11, 11) = 2.0998278684*000$
$L(10, 12) = 1.6253326234*000$	$L(10, 12) = 1.6253326234*000$	$L(10, 12) = 1.6253326234*000$	$L(10, 13) = 1.7640164096*000$
$L(11, 13) = 1.6984964548*000$	$L(11, 13) = 1.6984964548*000$	$L(11, 13) = 1.6984964548*000$	$L(11, 12) = 2.0998278684*000$
$L(15, 15) = 1.6059571898*000$	$L(15, 16) = 1.6840151393*000$	$L(15, 16) = 1.6840151393*000$	$L(15, 15) = 1.9246963970*000$
$L(14, 16) = 1.8038254829*000$	$L(14, 17) = 1.7356220718*000$	$L(14, 17) = 1.7356220718*000$	$L(14, 17) = 2.1347116555*000$
$L(15, 17) = 1.7356220718*000$	$L(15, 16) = 1.8038254829*000$	$L(15, 16) = 1.8038254829*000$	$L(15, 16) = 1.9246963970*000$
$L(19, 19) = 1.8121569756*000$	$L(19, 20) = 1.7305911512*000$	$L(19, 20) = 1.7305911512*000$	$L(19, 19) = 2.0337410913*000$
$L(18, 20) = 1.6648153737*000$	$L(18, 19) = 1.7527887570*000$	$L(18, 19) = 1.7527887570*000$	$L(18, 21) = 1.9235396529*000$
$L(19, 21) = 1.7527887570*000$	$L(19, 20) = 1.6648153737*000$	$L(19, 20) = 1.6648153737*000$	$L(19, 20) = 2.0337410913*000$

Double precision

HEXAGON DIAGONALS

SHORT DIAGONALS

LONG DIAGONALS

L(23, 27) = 1.8762154707+000	L(24, 26) = 1.8261775875+000	L(24, 27) = 2.1825821892+000
L(22, 24) = 1.671904214+000	L(25, 27) = 1.7390380748+000	L(22, 25) = 1.7519931590+000
L(23, 25) = 1.6403252463+000	L(22, 26) = 1.5724072538+000	L(23, 26) = 2.0221076356+000
L(29, 33) = 1.8480598095+000	L(30, 32) = 1.8102494716+000	L(30, 33) = 2.1996164761+000
L(28, 30) = 1.7236909642+000	L(31, 33) = 1.7665406473+000	L(28, 31) = 1.8031992531+000
L(29, 31) = 1.6224078098+000	L(28, 32) = 1.5730042771+000	L(29, 32) = 1.9627938005+000
L(35, 39) = 1.8203257148+000	L(36, 38) = 1.7846669993+000	L(36, 39) = 2.1993319147+000
L(34, 36) = 1.7554571144+000	L(37, 39) = 1.7926448449+000	L(34, 37) = 1.8592221520+000
L(35, 37) = 1.6218977181+000	L(34, 38) = 1.5774619230+000	L(35, 38) = 1.9131792323+000
L(41, 45) = 1.8472095644+000	L(42, 44) = 1.7851927921+000	L(42, 45) = 2.1288761553+000
L(40, 42) = 1.6811464250+000	L(43, 45) = 1.7521669296+000	L(40, 43) = 1.8327664192+000
L(41, 43) = 1.6850147433+000	L(40, 44) = 1.6089533782+000	L(41, 44) = 2.0157183828+000

DIHEDRAL ANGLES -- EXPRESSED IN DEG

ORDERED TRIPLET	DIHD ANGLE	ORDERED TRIPLET	DIHD ANGLE	ORDERED TRIPLET	DIHD ANGLE	ORDERED TRIPLET	DIHD ANGLE
51	1	1	177.8931859	1	51	1	177.5236569
7	2	1	176.0069605	7	1	2	175.9009437
27	3	2	174.3393406	27	2	3	174.3300897
33	4	3	173.1020330	33	3	4	173.1306885
39	5	4	172.4510274	39	4	5	172.4704151
23	9	23	175.0041836	25	11	29	172.1960550
11	25	29	173.8717247	31	45	35	171.1637432
45	31	35	172.8941251	37	14	37	170.8289418
13	41	41	171.7769133	41	13	41	173.1667210
19	16	43	171.9027898	19	43	16	172.4464363
							21

Execution time--5 sec

INITIAL GUESS

```

AG(1) THRU AG( 55) = 1.00-001
AG( 56) = -2.00-001
AG( 57) = -2.00-001
AG( 58) = -2.00-001
AG( 59) = -2.00-001
AG( 60) = -2.00-001
AG( 61) = -1.20+001

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FACE ANGLES -- EXPRESSED AS EXCESS OVER 120 DEG

7 ITERATIONS REQUIRED

INDEX	EXCESS (DEG)	INDEX	EXCESS (DEG)	INDEX	EXCESS (DEG)
1	5.98676773192+000	22	-8.88492924959+000	43	-1.59913825040+001
2	8.00649548287+000	23	6.77993811160+000	4	1.64306147699+001
3	8.19716652545+000	24	-2.27501549551+000	5	-6.20267213613+000
4	7.94687062124+000	25	-1.24915982574-001	6	-6.62089275338+000
5	7.66613340880+000	26	1.96837387216+001	7	9.0149376945+000
6	2.38880532959+001	27	-6.04976010867+000	8	5.84932253429-001
7	-1.40922087792+001	28	-1.0907276576+001	9	-1.32069198333+001
8	-3.81305992761+000	29	1.23242445698+001	50	1.14542286571+001
9	1.19224841177+001	30	5.34540186536-001	1	-1.05931096873+000
10	-1.57392662879+001	31	-1.64020357116+001	2	-6.98351955990+000
11	8.91636124820+000	32	1.56146853723+001	3	4.23749915053+000
12	5.08364979642+000	33	-1.13294476002+000	4	5.93788760096+000
13	-1.22607558014+001	34	-1.21777836926+001	5	-1.35867848800+001
14	1.99645210405+001	35	1.07190048889+001	6	-1.19735354638+001
15	-1.14614081231+001	36	3.42216597935+000	7	1.60129909657+001
16	-4.55300388433+000	37	-1.64451277879+001	8	-1.63943330509+001
17	1.20643029744+001	38	1.21344555340+001	9	1.58937412425+001
18	1.28939352456+001	39	2.13370893223+000	60	1.53322668176+001
19	-8.34766943971+000	40	-1.24017713962+001	1	-1.20000000000+001
20	-1.69626194250-001	41	8.0998166933+000		
21	4.14065602239+000	42	6.0251727404+000		

Execution time--8 sec

HEXAGON DIAGONALS

SHORT DIAGONALS

LONG-DIAGONAL S

L(1,	1)	=	1.6183054397+000	L(1,	1)	=	1.6183054397+000	L(1,	1)	=	1.9023439479+000
L(56,	1)	=	1.7819081889+000	L(56,	1)	=	1.7819081889+000	L(56,	1)	=	2.1751967937+000
L(1,	56)	=	1.7819081889+000	L(56,	1)	=	1.7819081889+000	L(1,	1)	=	1.9023439479+000
L(2,	2)	=	1.5758819050+000	L(2,	2)	=	1.5758819050+000	L(2,	2)	=	1.8663878960+000
L(57,	2)	=	1.7976377868+000	L(57,	2)	=	1.7976377868+000	L(57,	2)	=	2.2315016125+000
L(2,	57)	=	1.7976377868+000	L(57,	2)	=	1.7976377868+000	L(2,	2)	=	1.8663878960+000
L(3,	3)	=	1.5717749493+000	L(3,	3)	=	1.5717749493+000	L(3,	3)	=	1.8629214936+000
L(58,	3)	=	1.7990939560+000	L(58,	3)	=	1.7990939560+000	L(58,	3)	=	2.2367390625+000
L(3,	58)	=	1.7990939560+000	L(58,	3)	=	1.7990939560+000	L(3,	3)	=	1.8629214936+000
L(4,	4)	=	1.5771626138+000	L(4,	4)	=	1.5771626138+000	L(4,	4)	=	1.8674693867+000
L(59,	4)	=	1.7971814055+000	L(59,	4)	=	1.7971814055+000	L(59,	4)	=	2.2298610404+000
L(4,	59)	=	1.7971814055+000	L(59,	4)	=	1.7971814055+000	L(4,	4)	=	1.8674693867+000
L(5,	5)	=	1.5831697166+000	L(5,	5)	=	1.5831697166+000	L(5,	5)	=	1.8725454204+000
L(60,	5)	=	1.7950260467+000	L(60,	5)	=	1.7950260467+000	L(60,	5)	=	2.2221185084+000
L(5,	60)	=	1.7950260467+000	L(60,	5)	=	1.7950260467+000	L(5,	5)	=	1.8725454204+000
L(7,	7)	=	1.9015083557+000	L(8,	8)	=	1.8265402203+000	L(8,	7)	=	2.1149897141+000
L(6,	8)	=	1.5963019740+000	L(9,	7)	=	1.697829127+000	L(6,	9)	=	1.7165976988+000
L(7,	9)	=	1.6978229127+000	L(6,	8)	=	1.5963019740+000	L(7,	8)	=	2.1149897141+000
L(11,	11)	=	1.5788190923+000	L(12,	12)	=	1.6153547635+000	L(12,	11)	=	1.8842380267+000
L(10,	12)	=	1.804541640+000	L(13,	11)	=	1.7746953302+000	L(10,	13)	=	2.203246934+000
L(11,	13)	=	1.7746953302+000	L(10,	12)	=	1.8045416400+000	L(11,	12)	=	1.8842380267+000
L(15,	15)	=	1.8791733642+000	L(16,	16)	=	1.8275471076+000	L(16,	15)	=	2.1057725058+000
L(14,	16)	=	1.6235413963+000	L(17,	15)	=	1.6909618198+000	L(14,	17)	=	1.7482017967+000
L(15,	17)	=	1.6909618198+000	L(14,	16)	=	1.6235413963+000	L(15,	16)	=	2.1057725058+000
L(19,	19)	=	1.8333812072+000	L(20,	20)	=	1.7670463659+000	L(20,	19)	=	2.0590457983+000
L(18,	20)	=	1.6546743496+000	L(21,	19)	=	1.7305686427+000	L(18,	21)	=	1.8674349597+000
L(19,	21)	=	1.7305686427+000	L(18,	20)	=	1.6546743496+000	L(19,	20)	=	2.0590457983+000
L(23,	23)	=	1.6493890956+000	L(24,	24)	=	1.7309596811+000	L(24,	23)	=	1.9634220185+000
L(22,	24)	=	1.7881516652+000	L(25,	23)	=	1.7118575221+000	L(22,	25)	=	2.0656900677+000
L(23,	25)	=	1.7118575221+000	L(22,	24)	=	1.7881516652+000	L(23,	24)	=	1.9634220185+000

L(27, 31) = 1.8774902033+000	L(28, 30) = 1.8293853757*000	L(28, 31) = 2.1847907592+000
L(26, 28) = 1.6768679709+000	L(29, 31) = 1.7366966892*000	L(26, 29) = 1.7466733471*000
L(27, 29) = 1.6373953190+000	L(26, 30) = 1.5716918141+000	L(27, 30) = 2.0238517948+000
L(33, 37) = 1.85518379981+000	L(34, 36) = 1.8178832963*000	L(34, 37) = 2.2044888039+000
L(32, 34) = 1.7220795081+000	L(35, 37) = 1.7611380843+000	L(32, 35) = 1.7904685253+000
L(33, 35) = 1.6162081970+000	L(32, 36) = 1.5712265884*000	L(33, 36) = 1.9680087938+000
L(39, 43) = 1.8280441429+000	L(40, 42) = 1.7983511100*000	L(40, 43) = 2.2052819598+000
L(38, 40) = 1.7503696046+000	L(41, 43) = 1.7822124648*000	L(38, 41) = 1.8373201905+000
L(39, 41) = 1.6139023645+000	L(38, 42) = 1.5761141005*000	L(39, 42) = 1.9265614689+000
L(45, 49) = 1.8571700199+000	L(46, 48) = 1.8052827928*000	L(46, 49) = 2.1399831050+000
L(44, 46) = 1.6754119639+000	L(47, 49) = 1.7371327173*000	L(44, 47) = 1.8003182014+000
L(45, 47) = 1.6714144952+000	L(44, 48) = 1.6055629391*000	L(45, 48) = 2.0312301928+000
L(51, 55) = 1.8231958346+000	L(52, 54) = 1.7678374179+000	L(52, 55) = 2.1496001362+000
L(50, 52) = 1.7227327077+000	L(53, 55) = 1.7815206310*000	L(50, 53) = 1.8754463446+000
L(51, 53) = 1.6679303849+000	L(50, 54) = 1.6016008945+000	L(51, 54) = 1.9558652298+000

DIHEDRAL ANGLES -- EXPRESSED IN DEG

ORDERED TRIPLET	DIHD ANGLE	ORDERED TRIPLET	DIHD ANGLE	ORDERED TRIPLET	DIHD ANGLE	ORDERED TRIPLET	DIHD ANGLE			
61	1	1	178.2143907	1	61	1	177.9011128			
7	2	1	176.6123236	7	1	176.5210148	2	31	3	174.0252045
31	3	2	175.1721086	31	2	175.1594635	3	37	4	172.6405181
37	4	3	174.0358028	37	3	174.0562503	4	43	5	171.869814
43	5	4	173.3071318	43	4	173.3326423	5	10	5	171.5027346
10	5	5	173.0683469	9	27	174.7783563	27	9	27	175.7507923
29	15	33	173.2665145	15	33	173.7825807	15	29	33	174.7539063
35	49	39	172.2166472	49	39	173.1200208	49	35	39	173.8451821
41	55	12	171.6997947	55	12	172.9260623	55	41	12	173.1986070
17	45	45	172.7445142	45	17	174.1184555	47	19	51	172.1628902
19	51	47	172.6235384	19	47	173.4546112	53	22	53	171.9917682
22	53	53	172.9079820	21	24	172.4731588	24	21	24	172.9648888

Execution time--8 sec

INITIAL GUESS

```

AG(1) THRU AG( 65) = 1.00..001
AG( 66) = -2.00-001
AG( 67) = -2.00-001
AG( 68) = -2.00-001
AG( 69) = -2.00-001
AG( 70) = -2.00-001
AG( 71) = -2.00-001
AG( 72) = -1.00-001
AG( 73) = -1.20+001

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FACE ANGLES -- EXPRESSED AS EXCESS OVER 120 DEG

7 ITERATIONS REQUIRED

INDEX	EXCESS (DEG)	INDEX	EXCESS (DEG)	INDEX	EXCESS (DEG)
1	5.99021994866+000	26	-1.02834760278+001	51	1.07656237002+001
2	8.02979379423+000	2/	1.26185522063+001	52	*5.58752024410-001
3	8.27333182009+000	28	3.62649141008-001	53	-1.34579606878+001
4	8.08446354902+000	29	-1.64510950927+001	54	1.32857607470+001
5	7.78390778458+000	30	1.59994114854+001	55	*2.03587801287+000
6	7.63870192020+000	31	-1.23744817599+000	56	*8.27397710981+000
7	4.28392557015+000	32	-1.25960503541+001	57	7.02778370141+000
8	2.39172289330+001	33	1.14002230900+001	58	3.89764198689+000
9	-1.409372582281+001	34	3.02054165579+000	59	-1.39013313126+001
10	-3.87094359865+000	35	-1.6586677011+001	60	1.0294734061+001
11	1.20111747266+001	36	1.28874560717+001	61	*3.81131362499+000
12	2.04040196091+001	37	1.78337678236+000	62	-2.29943186426+000
13	-1.15848006219+001	38	-1.30160510721+001	63	1.97018362938+000
14	-5.07423534290+000	39	9.31984220930+000	64	3.932979793862+000
15	1.29140523204+001	40	5.19128560474+000	65	*9.88220948486+000
16	-1.39362084810+001	41	-1.61659095960+001	66	*1.19804398973+001
17	8.60569035207+000	42	1.00608640709+001	67	*1.60595875885+001
18	2.51475959181+000	43	4.25617891487+000	68	*1.65466636402+001
19	-8.30469140679+000	44	-1.28249080915+001	69	-1.61689270980+001
20	1.49238384110+001	45	6.79626819391+000	70	-1.55678155692+001
21	-9.04220344520+000	46	7.47100370173+000	71	-1.52774038404+001
22	-2.06298863008+000	47	-1.57594067889+001	72	*4.28392557015+000
23	7.28654573958+000	48	1.74395831820+001	73	-1.20000000000+001
24	1.98247974998+001	49	-6.58850093423+000		
25	-6.07142772670+000	50	-7.59999323586+000		

HEXAGON DIAGONALS

SHORT DIAGONALS

L(1,	1)	=	1.6182346281+000	L(1,	=	1.6182346281+000	L(1,	=	1.9022837096+000		
L(66,	1)	=	1.7819355484+000	L(66,	1)	=	1.7819355484+000	L(66,	1)	=	2.1752942986+000
L(1,	66)	=	1.7819355484+000	L(66,	1)	=	1.7819355484+000	L(1,	1)	=	1.9022837096+000
L(2,	2)	=	1.5753810065+000	L(2,	2)	=	1.5753810065+000	L(2,	2)	=	1.8659649824+000
L(67,	2)	=	1.7978159847+000	L(67,	2)	=	1.7978159847+000	L(67,	2)	=	2.2321423150+000
L(2,	67)	=	1.7978159847+000	L(67,	2)	=	1.7978159847+000	L(2,	2)	=	1.8659649824+000
L(3,	3)	=	1.5701295203+000	L(3,	3)	=	1.5701295203+000	L(3,	3)	=	1.8615334299+000
L(68,	3)	=	1.7996742441+000	L(68,	3)	=	1.7996742441+000	L(68,	3)	=	2.2388273849+000
L(3,	68)	=	1.7996742441+000	L(68,	3)	=	1.7996742441+000	L(3,	3)	=	1.8615334299+000
L(4,	4)	=	1.5742046197+000	L(4,	4)	=	1.5742046197+000	L(4,	4)	=	1.8649718992+000
L(69,	4)	=	1.7982338365+000	L(69,	4)	=	1.7982338365+000	L(69,	4)	=	2.2336449308+000
L(4,	69)	=	1.7982338365+000	L(69,	4)	=	1.7982338365+000	L(4,	4)	=	1.8649718992+000
L(5,	5)	=	1.5806542472+000	L(5,	5)	=	1.5806542472+000	L(5,	5)	=	1.8704191640+000
L(70,	5)	=	1.7959315716+000	L(70,	5)	=	1.7959315716+000	L(70,	5)	=	2.2253702097+000
L(5,	70)	=	1.7959315716+000	L(70,	5)	=	1.7959315716+000	L(5,	5)	=	1.8704191640+000
L(6,	6)	=	1.5837546485+000	L(6,	6)	=	1.5837546485+000	L(6,	6)	=	1.8730399854+000
L(71,	6)	=	1.7948148636+000	L(71,	6)	=	1.7948148636+000	L(71,	6)	=	2.2213603945+000
L(6,	71)	=	1.7948148636+000	L(71,	6)	=	1.7948148636+000	L(6,	6)	=	1.8730399854+000
L(72,	72)	=	1.7682161991+000	L(7,	7)	=	1.6934650077+000	L(7,	72)	=	1.9986025766+000
L(7,	72)	=	1.6934650077+000	L(72,	7)	=	1.7682161991+000	L(7,	72)	=	1.9986025766+000
L(72,	72)	=	1.7682161991+000	L(7,	7)	=	1.6934650077+000	L(72,	7)	=	1.9986025766+000
L(9,	9)	=	1.9016661219+000	L(10,	10)	=	1.8271702348+000	L(10,	9)	=	2.1153410445+000
L(8,	10)	=	1.5962909391+000	L(11,	9)	=	1.6972887379+000	L(8,	11)	=	1.7156574837+000
L(9,	11)	=	1.6972887379+000	L(8,	10)	=	1.5962909391+000	L(9,	10)	=	2.1153410445+000
L(13,	13)	=	1.8817853010+000	L(14,	14)	=	1.8335214814+000	L(14,	13)	=	2.1095719407+000
L(12,	14)	=	1.622282808+000	L(15,	15)	=	1.6860863849+000	L(12,	15)	=	1.7378504830+000
L(13,	15)	=	1.6860863849+000	L(12,	14)	=	1.622282808+000	L(13,	14)	=	2.1095719407+000
L(17,	17)	=	1.5979408166+000	L(18,	16)	=	1.6550955732+000	L(18,	17)	=	1.9091214660+000
L(16,	18)	=	1.8021971094+000	L(19,	17)	=	1.7535774012+000	L(16,	19)	=	2.1623572045+000
L(17,	19)	=	1.7535774012+000	L(16,	18)	=	1.8021971094+000	L(17,	18)	=	1.9091214660+000
L(21,	21)	=	1.8472499672+000	L(22,	22)	=	1.7920946345+000	L(22,	21)	=	2.0761615435+000
L(20,	22)	=	1.6478350568+000	L(23,	21)	=	1.7137681307+000	L(20,	23)	=	1.8268756170+000
L(21,	23)	=	1.7137681307+000	L(20,	22)	=	1.6478350568+000	L(21,	22)	=	2.0761615435+000

Execution time—9 sec

SHORT DIAGONALS

 $L(25, 29) = 1.8783371954+0.000$ $L(24, 26) = 1.6766619364+0.000$ $L(25, 27) = 1.6354612575+0.000$ $L(31, 35) = 1.8543638613+0.000$ $L(30, 32) = 1.7211512557+0.000$ $L(31, 33) = 1.6118973736+0.000$ $L(37, 41) = 1.8333360166+0.000$ $L(36, 38) = 1.7474033273+0.000$ $L(37, 39) = 1.6075470694+0.000$ $L(43, 47) = 1.8130642566+0.000$ $L(42, 44) = 1.7679898562+0.000$ $L(43, 45) = 1.6095295806+0.000$ $L(49, 53) = 1.8636332986+0.000$ $L(48, 50) = 1.6717249013+0.000$ $L(49, 51) = 1.6619690042+0.000$ $L(55, 59) = 1.8361031912+0.000$ $L(54, 56) = 1.7140120666+0.000$ $L(55, 57) = 1.6553964584+0.000$ $L(61, 65) = 1.8128329123+0.000$ $L(60, 62) = 1.6978390215+0.000$ $L(61, 63) = 1.7116371267+0.000$ $L(26, 28) = 1.8314553122+0.000$ $L(27, 29) = 1.7352068395+0.000$ $L(24, 28) = 1.5711621474+0.000$ $L(32, 34) = 1.8228081556+0.000$ $L(33, 35) = 1.7578052678+0.000$ $L(30, 34) = 1.5696968402+0.000$ $L(30, 38) = 1.8075667759+0.000$ $L(39, 41) = 1.7755607711+0.000$ $L(36, 40) = 1.5742371043+0.000$ $L(44, 46) = 1.7882793091+0.000$ $L(45, 47) = 1.7935215918+0.000$ $L(42, 46) = 1.5786032826+0.000$ $L(50, 52) = 1.8182223763+0.000$ $L(51, 53) = 1.7271542053+0.000$ $L(48, 52) = 1.6029465229+0.000$ $L(56, 58) = 1.7900850395+0.000$ $L(57, 59) = 1.7650557777+0.000$ $L(54, 58) = 1.5983068096+0.000$ $L(62, 64) = 1.7499870631+0.000$ $L(63, 65) = 1.7658402672+0.000$ $L(60, 64) = 1.6394823998+0.000$ $L(61, 64) = 1.9946731041+0.000$

DIHEDRAL ANGLES -- EXPRESSED IN DEG

ORDERED TRIPLET

DIHD ANGLE

ORDERED TRIPLET

DIHD ANGLE

LONG DIAGONALS

 $L(26, 29) = 2.1862754708+0.000$ $L(24, 27) = 1.7431783422+0.000$ $L(25, 28) = 2.0249351187+0.000$ $L(32, 35) = 2.2080234263+0.000$ $L(30, 33) = 1.781977520+0.000$ $L(31, 34) = 1.9710183378+0.000$ $L(38, 41) = 2.2103871304+0.000$ $L(36, 39) = 1.8219190699+0.000$ $L(37, 40) = 1.9344059746+0.000$ $L(44, 47) = 2.2073427418+0.000$ $L(42, 45) = 1.8626507107+0.010$ $L(43, 46) = 1.9005654974+0.000$ $L(50, 53) = 2.1474971732+0.000$ $L(48, 51) = 1.7784158319+0.000$ $L(49, 52) = 2.0408938742+0.000$ $L(56, 59) = 2.1601456614+0.000$ $L(54, 57) = 1.8396555892+0.000$ $L(55, 58) = 1.9766914190+0.000$ $L(62, 65) = 2.0886622389+0.000$ $L(60, 63) = 1.906216949+0.000$ $L(61, 64) = 1.9946731041+0.000$ $L(29) = 1.9 2 1 9 2 176.4409647$ $L(24, 27) = 2 29 3 29 3 174.8360480$ $L(25, 28) = 3 35 4 35 4 173.5726466$ $L(32, 35) = 4 41 5 41 5 172.7022279$ $L(30, 33) = 5 47 6 47 6 172.2657670$ $L(31, 34) = 11 25 25 11 25 175.4934951$ $L(36, 39) = 13 31 27 13 31 174.5870394$ $L(37, 40) = 53 37 33 53 37 173.9052380$ $L(44, 47) = 59 43 39 59 43 173.5644775$ $L(42, 45) = 45 16 45 45 16 173.6021541$ $L(43, 46) = 49 174.8847080 51 21 55 172.923688$ $L(48, 51) = 55 174.2704198 57 65 18 172.6307936$ $L(50, 53) = 65 173.7393809 23 61 61 173.0433678$ $L(51, 54) = 63 72 63 72 63 63 173.3921892$

Execution time=9 sec

POLYHEDRON WITH 1692 FACES

Double precision

INITIAL GUESS

AG(1) THRU AG(78)	=	1.00-001
AG(79)	=	-2.00-001
AG(80)	=	-2.00-001
AG(81)	=	-2.00-001
AG(82)	=	-2.00-001
AG(83)	=	-2.00-001
AG(84)	=	-2.00-001
AG(85)	=	-1.20+001

FACE ANGLES -- EXPRESSED AS EXCESS OVER 120 DEG

7 ITERATIONS REQUIRED

INDEX	EXCESS (DEG)	INDEX	EXCESS (DEG)	INDEX	EXCESS (DEG)	INDEX	EXCESS (DEG)
1	5.99259565651+000	30	-9.73142467703-002	59	59	-1.31383461213+000	
2	8.04599280394+000	31	1.99220769125+001	60	-1.36534950123+001		
3	8.32846888402+000	32	-6.08593649707+000	61	1.45732947280+001		
4	8.19573515471+000	33	-1.04150592983+001	62	-2.68081426614+000		
5	7.91352721588+000	34	1.28117211309+001	63	-9.24652037251+000		
6	7.68580301118+000	35	2.48401001468-001	64	9.02913270191+000		
7	2.39373192081+001	36	-1.64872032495+001	65	2.49278579235+000		
8	-1.40941001237+001	37	1.62669332115+001	66	-1.41678785836+001		
9	-3.91042951474+000	38	-1.30094756314+000	67	1.05767421696+001		
10	1.20717400689+001	39	-1.28905214115+001	68	1.25828836081+000		
11	-1.56629325062+001	40	1.18614166482+001	69	-9.35536660250+000		
12	8.58252210058+000	41	2.76406770782+000	70	5.26093221179+000		
13	5.83848381594+000	42	-1.67009485929+001	71	6.44993685854+000		
14	-1.31790793268+001	43	1.34233092655+001	72	-1.41905329983+001		
15	2.07053867370+001	44	1.57729715751+000	73	1.23982423117+001		
16	-1.16731777696+001	45	-1.34905659165+001	74	-4.90178991267+000		
17	-5.42141679517+000	46	1.017239066686+001	75	-4.02863064385+000		
18	1.34838023927+001	47	4.66228220000+000	76	5.29166070133+000		
19	1.63331082540+001	48	-1.63447133751+001	77	1.80885237942+000		
20	-9.52533045435+000	49	1.0909937408+001	78	-1.05683348359+001		
21	-3.40477206863+000	50	3.7328096248+000	79	-1.19851913130+001		
22	9.52709679196+000	51	-1.33362550933+001	80	-1.60919856079+001		
23	-1.08135027469+001	52	8.06302727249+000	81	-1.66569377680+001		
24	7.27603406689+000	53	6.50617149961+000	82	-1.63914703094+001		
25	1.70615201647+001	54	-1.58759183821+001	83	-1.58270544318+001		
26	-4.07979579017+000	55	1.81324234102+001	84	-1.5371606024+001		
27	7.79135114797+000	56	-6.8446283089+000	85	-1.20000000000+001		
28	-5.75203354629+000	57	-8.27663470605+000				
29	1.90501509569+000	58	1.19560237511+001				

Execution time--14 sec

HEXAGON DIAGONALS

SHORT DIAGONALS

L(1, 1) = 1.6181858944+000	L(1, 1) = 1.6181858944+000
L(79, 1) = 1.7819543754+000	L(79, 1) = 1.7819543754+000
L(1, 79) = 1.7819543754+000	L(79, 1) = 1.7819543754+000
L(2, 2) = 1.5750325848+000	L(2, 2) = 1.5750325848+000
L(80, 2) = 1.7979398397+000	L(80, 2) = 1.7979398397+000
L(2, 80) = 1.7979398397+000	L(80, 2) = 1.7979398397+000
L(3, 3) = 1.5689366410+000	L(3, 3) = 1.5689366410+000
L(81, 3) = 1.8000938262+000	L(81, 3) = 1.8000938262+000
L(3, 81) = 1.8000938262+000	L(81, 3) = 1.8000938262+000
L(4, 4) = 1.5718058452+000	L(4, 4) = 1.5718058452+000
L(82, 4) = 1.7990830431+000	L(82, 4) = 1.7990830431+000
L(4, 82) = 1.7990830431+000	L(82, 4) = 1.7990830431+000
L(5, 5) = 1.5778780670+000	L(5, 5) = 1.5778780670+000
L(83, 5) = 1.7969259759+000	L(83, 5) = 1.7969259759+000
L(5, 83) = 1.7969259759+000	L(83, 5) = 1.7969259759+000
L(6, 6) = 1.5827500708+000	L(6, 6) = 1.5827500708+000
L(84, 6) = 1.7951774111+000	L(84, 6) = 1.7951774111+000
L(6, 84) = 1.7951774111+000	L(84, 6) = 1.7951774111+000
L(8, 8) = 1.9017746877+000	L(9, 9) = 1.8275998323+000
L(7, 9) = 1.5962820866+000	L(10, 8) = 1.6969240968+000
L(8, 10) = 1.6969240968+000	L(7, 9) = 1.5962820866+000
L(12, 12) = 1.5796365820+000	L(13, 13) = 1.6058526260+000
L(11, 13) = 1.8020217353+000	L(14, 12) = 1.7807314843+000
L(12, 14) = 1.7807314843+000	L(11, 13) = 1.8020217353+000
L(16, 16) = 1.8835603279+000	L(17, 17) = 1.8374708072+000
L(15, 17) = 1.6213802034+000	L(18, 16) = 1.6828195961+000
L(16, 18) = 1.6828195961+000	L(15, 17) = 1.6213802034+000
L(20, 20) = 1.8565377724+000	L(21, 21) = 1.8091119767+000
L(19, 21) = 1.6430418403+000	L(22, 20) = 1.7015784517+000
L(20, 22) = 1.7015784517+000	L(19, 21) = 1.6430418403+000
L(24, 24) = 1.6301190625+000	L(25, 25) = 1.6953577630+000
L(23, 25) = 1.7920151769+000	L(26, 24) = 1.7335377857+000
L(24, 26) = 1.7335377857+000	L(23, 25) = 1.7920151769+000
L(28, 28) = 1.7959887372+000	L(29, 29) = 1.7312009561+000
L(27, 29) = 1.6796943127+000	L(30, 28) = 1.7484350964+000
L(28, 30) = 1.7484350964+000	L(27, 29) = 1.6796943127+000

LONG DIAGONALS

L(1, 1) = 1.9022427529+000	L(1, 1) = 1.9022427529+000
L(79, 1) = 2.1753613960+000	L(79, 1) = 2.1753613960+000
L(1, 79) = 1.9022427529+000	L(1, 79) = 1.9022427529+000
L(2, 2) = 1.8656708292+000	L(2, 2) = 1.8656708292+000
L(80, 2) = 2.325876670+000	L(80, 2) = 2.325876670+000
L(2, 80) = 1.8656708292+000	L(2, 80) = 1.8656708292+000
L(3, 3) = 1.8605273939+000	L(3, 3) = 1.8605273939+000
L(81, 3) = 2.2403377833+000	L(81, 3) = 2.2403377833+000
L(3, 81) = 1.8605273939+000	L(3, 81) = 1.8605273939+000
L(4, 4) = 1.8629475610+000	L(4, 4) = 1.8629475610+000
L(82, 4) = 2.366997958+000	L(82, 4) = 2.366997958+000
L(4, 82) = 1.8629475610+000	L(4, 82) = 1.8629475610+000
L(5, 5) = 1.8680736587+000	L(5, 5) = 1.8680736587+000
L(83, 5) = 2.2289429627+000	L(83, 5) = 2.2289429627+000
L(5, 83) = 1.8680736587+000	L(5, 83) = 1.8680736587+000
L(6, 6) = 1.8721906385+000	L(6, 6) = 1.8721906385+000
L(84, 6) = 2.226619375+000	L(84, 6) = 2.226619375+000
L(6, 84) = 1.8721906385+000	L(6, 84) = 1.8721906385+000
L(8, 8) = 2.1155810314+000	L(9, 8) = 2.1155810314+000
L(7, 9) = 1.7150138323+000	L(7, 10) = 1.7150138323+000
L(8, 10) = 2.1155810314+000	L(8, 9) = 2.1155810314+000
L(12, 12) = 1.8806019125+000	L(13, 12) = 1.8806019125+000
L(11, 13) = 2.093332892+000	L(11, 14) = 2.093332892+000
L(12, 14) = 1.8806019125+000	L(12, 13) = 1.8806019125+000
L(16, 16) = 2.1121049018+000	L(17, 16) = 2.1121049018+000
L(15, 17) = 1.7308374297+000	L(15, 17) = 1.7308374297+000
L(16, 18) = 2.1093547389+000	L(16, 17) = 2.1093547389+000
L(20, 20) = 2.0877463254+000	L(21, 20) = 2.0877463254+000
L(19, 21) = 1.7979834336+000	L(19, 22) = 1.7979834336+000
L(20, 22) = 2.0877463254+000	L(20, 21) = 2.0877463254+000
L(24, 24) = 1.9400090224+000	L(25, 24) = 1.9400090224+000
L(23, 25) = 2.1093547389+000	L(23, 26) = 2.1093547389+000
L(24, 26) = 1.9400090224+000	L(24, 25) = 1.9400090224+000
L(28, 28) = 2.0271204747+000	L(29, 28) = 2.0271204747+000
L(27, 29) = 1.9402173996+000	L(28, 29) = 1.9402173996+000
L(28, 30) = 2.0271204747+000	L(28, 29) = 2.0271204747+000

DIHEDRAL ANGLES -- EXPRESSED IN DEG			DIHEDRAL ANGLE			DIHEDRAL ANGLE			DIHEDRAL ANGLE		
ORDERED TRIPLET	DIHEDRAL ANGLE	DEG	ORDERED TRIPLET	DIHEDRAL ANGLE	DEG	ORDERED TRIPLET	DIHEDRAL ANGLE	DEG	ORDERED TRIPLET	DIHEDRAL ANGLE	DEG
85	1	1	178.6642633	1	178.42998404	1	1	1	1	8	2
8	2	1	177.46229482	2	177.3933897	2	2	2	3	36	3
36	3	2	176.33627263	3	176.34485749	3	3	3	42	4	175.4901136
42	4	3	175.4354315	4	175.4437814	4	4	4	48	5	174.3444980
48	5	4	174.7228996	5	174.7432650	5	5	5	54	6	173.4954352
54	6	5	174.2772771	6	174.2949513	6	6	6	11	6	172.9813549
11	6	6	174.1332353	10	176.0683568	10	32	32	10	32	176.8083043
34	16	38	174.8571860	16	175.2488621	16	34	34	38	38	176.0282349
40	60	44	173.8977756	60	174.5844825	60	40	40	44	44	175.2674212
46	66	50	173.2498264	66	174.1683969	66	46	46	50	50	174.6433617
52	72	13	172.9321694	72	174.0494586	72	52	52	13	13	174.2213556
18	56	56	174.3075581	56	175.5106524	56	20	20	62	62	173.6244024
20	62	58	173.9548008	20	174.9428911	20	58	58	64	68	173.2378074
78	68	64	173.8728218	78	174.4338330	78	23	23	70	70	173.184889
23	70	70	174.0541889	22	173.5912658	22	74	74	22	22	174.5445376
76	28	25	173.4501502	28	173.7909043	28	76	76	25	25	174.1391267
30	30	30	173.78992988								

Execution time=14 sec

POLYHEDRON WITH 1962 FACES
INITIAL GUESS

AG(1) THRU AG(91)	=	1.00-001
AG(92)	=	-2.00+001
AG(93)	=	-2.00+001
AG(94)	=	-2.00+001
AG(95)	=	-2.00+001
AG(96)	=	-2.00+001
AG(97)	=	-2.00+001
AG(98)	=	-2.00+001
AG(99)	=	-1.20+001

FACE ANGLES -- EXPRESSED AS EXCESS OVER 120 DEG
7 ITERATIONS REQUIRED

INDEX	EXCESS(DEG)	INDEX	EXCESS(DEG)	INDEX	EXCESS(DEG)
1	5.99427813026+000	34	-1.05077090653+001	67	-1.38058590366+001
2	8.05754251360+000	35	1.29570964574+001	68	1.54936432044+001
3	8.36886871194+000	36	1.69486562908+001	69	-3.11269090737+000
4	8.28342737071+000	37	-1.65139313966+001	70	-9.97550954808+000
5	8.03556584190+000	38	1.64577952635+001	71	1.04644722927+001
6	7.77494505978+000	39	-1.34154820264+000	72	1.52715614466+000
7	7.66249150644+000	40	-1.31020712920+001	73	-1.43270711863+001
8	2.39515533004+001	41	1.21832402619+001	74	1.20350667851+001
9	-1.40947510434+001	42	2.59342130922+000	75	3.75180053373-001
10	-3.93823647858+000	43	-1.67908373399+001	76	-1.02085379745+001
11	1.2114217435+001	44	1.38119573538+001	77	7.34263507692+000
12	2.09182647746+001	45	1.45255701072+000	78	4.87758868770+000
13	-1.17376399636+001	46	-1.38540438622+001	79	-1.44219326285+001
14	-5.66085822575+000	47	1.07772449514+001	80	1.41202314810+001
15	1.38787316041+001	48	4.31871284914+000	81	-5.67438432262+000
16	-1.43715486891+001	49	-1.45069078465+001	82	-5.35421252853+000
17	8.42110286126+000	50	1.15460720644+001	83	7.79927791852+000
18	3.85578543140+000	51	3.40667753681+000	84	1.81095395295-001
19	-1.0182278963+001	52	-1.37829762683+001	85	1.10720079436+001
20	1.73299698614+001	53	9.00358881447+000	86	9.79813038272+000
21	-9.8723492105+000	54	5.86409198268+000	87	-1.21784434216+000
22	-4.35306265772+000	55	-1.60374541301+001	88	-5.54683672126+000
23	1.11208552962+001	56	9.44893966551+000	89	3.44101836634+000
24	1.04438490224+001	57	5.17664099079+000	90	4.97289110455+000
25	-6.82621785127+000	58	-1.35382362691+001	91	-1.14473587902+001

HEXAGON DIAGONALS		SHORT DIAGONALS		LONG DIAGONALS	
L(26,	-1.10775233817+000	59	7.05980783279+000	92	-1.19885562605+001
27	3.39013714775+000	60	7.53868415871+000	93	-1.611508850272+001
28	-7.132435530623+000	61	-1.56858363787+001	94	-1.6377374239+001
29	5.42516675793+000	62	1.86214536958+001	95	-1.5668547414+001
30	-1.82013331708+000	63	-7.02038792853+000	96	-1.60711316838+001
31	-7.76315754790+002	64	-8.75323331740+000	97	-1.52498901196+001
32	1.99910795468+001	65	1.27835804269+001	98	-1.53249830129+001
33	-6.09602210520+000	66	-1.82546384026+000	99	-1.20000000000+001
L(1,	1) = 1.6181513796+000	L(1,	1) = 1.6181513796+000	L(1,	1) = 1.9022128922+000
L(92,	1) = 1.7819677082+000	L(92,	1) = 1.7819677082+000	L(92,	1) = 2.1754089130+000
L(1,	92) = 1.7819677082+000	L(92,	1) = 1.7819677082+000	L(1,	1) = 1.9022128922+000
L(2,	2) = 1.5747840872+000	L(2,	2) = 1.5747840872+000	L(2,	2) = 1.8654610479+000
L(93,	2) = 1.7980281249+000	L(93,	2) = 1.7980281249+000	L(93,	2) = 2.329051378+000
L(2,	93) = 1.7980281249+000	L(93,	2) = 1.7980281249+000	L(2,	2) = 1.8654610479+000
L(3,	3) = 1.5680616764+000	L(3,	3) = 1.5680616764+000	L(3,	3) = 1.8597896174+000
L(94,	3) = 1.8004009964+000	L(94,	3) = 1.8004009964+000	L(94,	3) = 2.2414437478+000
L(3,	94) = 1.8004009964+000	L(94,	3) = 1.8004009964+000	L(3,	3) = 1.8597896174+000
L(4,	4) = 1.5699112138+000	L(4,	4) = 1.5699112138+000	L(4,	4) = 1.8613493007+000
L(95,	4) = 1.7997511004+000	L(95,	4) = 1.7997511004+000	L(95,	4) = 2.391040234+000
L(4,	95) = 1.7997511004+000	L(95,	4) = 1.7997511004+000	L(4,	4) = 1.8613493007+000
L(5,	5) = 1.5752568709+000	L(5,	5) = 1.5752568709+000	L(5,	5) = 1.8658601795+000
L(96,	5) = 1.7978601210+000	L(96,	5) = 1.7978601210+000	L(96,	5) = 2.2323010146+000
L(5,	96) = 1.7978601210+000	L(96,	5) = 1.7978601210+000	L(5,	5) = 1.8658601795+000
L(6,	6) = 1.580459114+000	L(6,	6) = 1.580459114+000	L(6,	6) = 1.8705811384+000
L(97,	6) = 1.7958627271+000	L(97,	6) = 1.7958627271+000	L(97,	6) = 2.2251229345+000
L(6,	97) = 1.7958627271+000	L(97,	6) = 1.7958627271+000	L(6,	6) = 1.8705811384+000
L(7,	7) = 1.5832473951+000	L(7,	7) = 1.5832473951+000	L(7,	7) = 1.87264110953+000
L(98,	7) = 1.7949980152+000	L(98,	7) = 1.7949980152+000	L(98,	7) = 2.2220178746+000
L(7,	98) = 1.7949980152+000	L(98,	7) = 1.7949980152+000	L(7,	7) = 1.87264110953+000
L(9,	9) = 1.9018515719+000	L(10,	10) = 1.8279022721+000	L(10,	9) = 2.1197501765+000
L(8,	10) = 1.5962752420+000	L(11,	9) = 1.6966671866+000	L(8,	11) = 1.7145595044+000
L(9,	11) = 1.6966671866+000	L(8,	10) = 1.5962752420+000	L(9,	10) = 2.1197501765+000

HEXAGON DIAGONALS

SHORT DIAGONALS

L(13, 13) = 1.8848063109+000	L(14, 14) = 1.8401816809+000	L(14, 13) = 2.1138557295+000
L(12, 14) = 1.6207212416+000	L(15, 13) = 1.6805575802+000	L(12, 15) = 1.7259352345+000
L(13, 15) = 1.6805575802+000	L(12, 14) = 1.6207212416+000	L(13, 14) = 2.1138557295+000
L(17, -17) = 1.5933600122+000	L(18, 18) = 1.6364777713+000	L(18, 17) = 1.8993415284+000
L(16, 18) = 1.8007978142+000	L(19, 17) = 1.7647121187+000	L(16, 19) = 2.1790472681+000
L(17, 19) = 1.7647121187+000	L(16, 18) = 1.8007978142+000	L(17, 18) = 1.8993415284+000
L(21, 21) = 1.8629381211+000	L(22, 22) = 1.8207962514+000	L(22, 21) = 2.0957172394+000
L(20, 22) = 1.6395809509+000	L(23, 21) = 1.6928227285+000	L(20, 23) = 1.7773318398+000
L(21, 23) = 1.6928227285+000	L(20, 22) = 1.6395809509+000	L(21, 22) = 2.0957172394+000
L(25, 25) = 1.8161683146+000	L(26, 26) = 1.7608730913+000	L(26, 25) = 2.0489123735+000
L(24, 26) = 1.6694437895+000	L(27, 25) = 1.7310833021+000	L(24, 27) = 1.8925694520+000
L(25, 27) = 1.7310833021+000	L(24, 26) = 1.6694437895+000	L(25, 26) = 2.0489123735+000
L(29, 29) = 1.6664947657+000	L(30, 30) = 1.7313729469+000	L(30, 29) = 1.9711225110+000
L(28, 30) = 1.7774358810+000	L(31, 29) = 1.7159493303+000	L(28, 31) = 2.0529606516+000
L(29, 31) = 1.7159493303+000	L(28, 30) = 1.7774358810+000	L(29, 30) = 1.9711225110+000
L(33, 37) = 1.8793319863+000	L(34, 36) = 1.8338214337+000	L(34, 37) = 2.1880365100+000
L(32, 34) = 1.6764277855+000	L(35, 37) = 1.7335279618+000	L(32, 35) = 1.7391179786+000
L(33, 35) = 1.6332054526+000	L(32, 36) = 1.5704833182+000	L(33, 36) = 2.0261286092+000
L(39, 43) = 1.8573460228+000	L(40, 42) = 1.8283893819+000	L(40, 43) = 2.2125079334+000
L(38, 40) = 1.7202251632+000	L(41, 43) = 1.7542371853+000	L(38, 41) = 1.7720315129+000
L(39, 41) = 1.6066534186+000	L(38, 42) = 1.5674862427+000	L(39, 42) = 1.9740051827+000
L(45, 49) = 1.8397248635+000	L(46, 48) = 1.8183103417+000	L(46, 49) = 2.2179852076+000
L(44, 46) = 1.7445872664+000	L(47, 49) = 1.7684998294+000	L(44, 47) = 1.8030840299+000
L(45, 47) = 1.5988027967+000	L(44, 48) = 1.570592181+000	L(45, 48) = 1.9418834979+000
L(51, 55) = 1.8238542026+000	L(52, 54) = 1.8051975336+000	L(52, 55) = 2.2154839552+000
L(50, 52) = 1.7610099564+000	L(53, 55) = 1.7809349106+000	L(50, 53) = 1.8341481000+000
L(51, 53) = 1.5995476963+000	L(50, 54) = 1.5756189664+000	L(51, 54) = 1.9174083379+000
L(57, 61) = 1.8085299657+000	L(58, 60) = 1.7903342401+000	L(58, 61) = 2.2119510795+000
L(56, 58) = 1.7754431138+000	L(59, 61) = 1.7940439991+000	L(56, 59) = 1.8648604236+000
L(57, 59) = 1.6021082450+000	L(56, 60) = 1.5793913686+000	L(57, 60) = 1.8928833368+000

LONG DIAGONALS

L(63, 67) = 1.8710203832+000	L(64, 66) = 1.8326107100+000	L(64, 67) = 2.1564465238+000
L(62, 64) = 1.6675752184+000	L(65, 67) = 1.7159015370+000	L(62, 65) = 1.7528245080+000
L(63, 65) = 1.6506871167+000	L(62, 66) = 1.5993079162+000	L(63, 66) = 2.0512038128+000
L(69, 73) = 1.8510389953+000	L(70, 72) = 1.8160266597+000	L(70, 73) = 2.1738091407+000
L(68, 70) = 1.7042518398+000	L(71, 73) = 1.7452235541+000	L(68, 71) = 1.7952913698+000
L(69, 71) = 1.6385492201+000	L(68, 72) = 1.5930907411+000	L(69, 72) = 1.994033983+000
L(75, 79) = 1.8273397651+000	L(76, 78) = 1.7925290302+000	L(76, 79) = 2.1785527200+000
L(74, 76) = 1.7353155820+000	L(77, 79) = 1.7730341375+000	L(74, 77) = 1.8459148978+000
L(75, 77) = 1.6362137455+000	L(74, 78) = 1.5928283684+000	L(75, 78) = 1.9506425517+000
L(81, 85) = 1.8418286174+000	L(82, 84) = 1.7960496071+000	L(82, 85) = 2.1169353655+000
L(80, 82) = 1.6804295794+000	L(83, 85) = 1.7336289994+000	L(80, 83) = 1.8289305203+000
L(81, 83) = 1.6834531583+000	L(80, 84) = 1.6275009019+000	L(81, 84) = 2.0333742921+000
L(87, 91) = 1.8111237557+000	L(88, 90) = 1.7612939966+000	L(88, 91) = 2.1243202426+000
L(86, 88) = 1.7213254960+000	L(89, 91) = 1.7738031452+000	L(86, 89) = 1.8957320986+000
L(87, 89) = 1.6816356632+000	L(86, 90) = 1.6236845793+000	L(87, 90) = 1.9664907431+000

DIHEDRAL ANGLES -- EXPRESSED IN DEG

ORDERED TRIPLET	DIHD ANGLE	DIHD ANGLE	ORDERED TRIPLET	DIHD ANGLE	DIHD ANGLE
99	1	178.8257813	1	99	178.6196864
9	2	177.7689980	9	1	177.7075270
37	3	176.7939581	37	2	176.7822076
43	4	175.9605071	43	3	175.9652740
49	5	175.2878839	49	4	175.3039105
55	6	174.8164102	55	5	174.8347866
61	7	174.5783987	61	6	174.5866468
32	11	177.1908661	35	13	175.4511969
13	35	176.4958156	41	67	174.5580433
67	41	175.8034526	47	73	173.9051836
73	47	175.2115795	53	79	173.5175196
79	53	174.7742197	59	16	173.3909597
15	63	174.9243149	63	15	176.0280606
21	69	174.5341544	21	65	175.5044743
85	75	174.3513897	85	71	175.0205224
91	77	174.3955680	91	77	174.6353256
81	23	175.1350119	83	25	173.8978507
25	83	174.7577278	89	28	173.8379687
27	30	174.1417910	30	27	174.4520038

INITIAL GUESS

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AG(1) THRU AG(104) = 1.00-001
AG(105) = -2.00-001
AG(106) = -2.00-001
AG(107) = -2.00-001
AG(108) = -2.00-001
AG(109) = -2.00-001
AG(110) = -2.00-001
AG(111) = -2.00-001
AG(112) = -1.00-001
AG(113) = -1.20+001

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FACE ANGLES -- EXPRESSED AS EXCESS OVER 120 DEG

7 ITERATIONS REQUIRED

INDEX	EXCESS(DEG)	INDEX	EXCESS(DEG)	INDEX	EXCESS(DEG)
1	5.99549934034+000	39	1.65970807513+001	77	-1.08993137733+001
2	8.06596322613+000	40	*1.36869619455+000	78	8.91167300627+000
3	8.39888941241+000	41	*1.32570774993+001	79	3.74701938445+000
4	8.35192879370+000	42	1.24139943499+001	80	-1.46414757689+001
5	8.14240069687+000	43	2.47563487389+000	81	9.94465843435+000
6	7.88036092164+000	44	*1.68609362812+001	82	2.79960552730+000
7	7.69674270463+000	45	1.40989878447+001	83	*1.08743105735+001
8	3.44047521534+000	46	1.37448191462+000	84	5.90077209645+000
9	2.39618882164+001	47	-1.41325820424+001	85	6.76957532985+000
10	*1.40952490126+001	48	1.12156384381+001	86	-1.45403008145+001
11	-3.9583483497+000	49	4.08919468036+000	87	1.53819156862+001
12	1.21453065083+001	50	*1.66457208354+001	88	-6.22753657655+000
13	-1.5612270174+001	51	1.20275163216+001	89	-6.35567422469+000
14	8.37068028221+000	52	3.20252070268+000	90	9.66847646256+000
15	6.31085323378+000	53	*1.41603336603+001	91	-1.01051708823+000
16	-1.37507966854+001	54	9.70569408210+000	92	-1.4566642593+001
17	2.10725067802+001	55	5.43408906971+000	93	1.17157785007+001
18	-1.17854582248+001	56	*1.62094865158+001	94	-2.25394861044+000
19	-5.83108735418+000	57	1.01287144539+001	95	-6.73654955551+000
20	1.41605843777+001	58	4.73387160938+000	96	6.03246082723+000
21	1.80499653503+001	59	*1.38959367410+001	97	3.16535822424+000
22	-1.01211235561+001	60	8.01211235561+000	98	-1.19230993862+001

23	-5.03045782031+000
24	1.22666237716+001
25	-1.2020047914+001
26	7.50334416207+000
27	1.89340756066+000
28	-6.77349865409+000
29	1.25217550978+001
30	-7.62146306013+000
31	-1.71990965862+000
32	6.16099033975+000
33	2.00412260152+001
34	-6.10325073976+000
35	-1.05746825844+001
36	1.30574077981+001
37	1.13221494110-001
38	-1.65339219833+001
61	6.80879236450+000
62	-1.57875540430+001
63	1.89752731684+001
64	-7.14505663038+000
65	-9.09643379737+000
66	1.33728623845+001
67	-2.18197594813+000
68	-1.394669170+001
69	1.616396888731+001
70	*3.40848881268+000
71	-1.05234109134+001
72	1.15036384305+001
73	8.55769772897-001
74	-1.45914773504+001
75	1.31216603445+001
76	-2.39563193068-001
99	8.25594379992+000
100	-3.18805147955+000
101	-1.82172929334+000
102	1.61142297989+000
103	3.28471420968+000
104	-8.14230021660+000
105	-1.19909986807+001
106	-1.61319264523+001
107	-1.6797788248+001
108	-1.67038575874+001
109	-1.62848013937+001
110	-1.57607218433+001
111	-1.53934854093+001
112	-3.44047521534+000
113	-1.200000000000+001

HEXAGON DIAGONALS

SHORT DIAGONALS

L(1, 1) = 1.6181263264+000	L(-1, 1) = 1.6181263264+000	L(1, 1) = 1.9021915803+000
L(105, 1) = 1.7819773854+000	L(105, 1) = 1.7819773854+000	L(105, 105) = 2.1754434022+000
L(-1,105) = 1.7819773854+000	L(105, 1) = 1.7819773854+000	L(-1, 1) = 1.9021915803+000
L(2, 2) = 1.5746028711+000	L(-2, 2) = 1.5746028711+000	L(2, 2) = 1.8653080716+000
L(106, 2) = 1.7980924807+000	L(106, 2) = 1.7980924807+000	L(106, 106) = 2.2331365692+000
L(-2,106) = 1.7980924807+000	L(106, 2) = 1.7980924807+000	L(-2, 2) = 1.8653080716+000
L(3, 3) = 1.5674109943+000	L(-3, 3) = 1.5674109943+000	L(3, 3) = 1.8592410347+000
L(107, 3) = 1.8006291065+000	L(107, 3) = 1.8006291065+000	L(107, 107) = 2.2422651790+000
L(-3,107) = 1.8006291065+000	L(107, 3) = 1.8006291065+000	L(-3, 3) = 1.8592410347+000
L(4, 4) = 1.5684286499+000	L(-4, 4) = 1.5684286499+000	L(4, 4) = 1.8600990376+000
L(108, 4) = 1.8002722251+000	L(108, 4) = 1.8002722251+000	L(108, 108) = 2.2409800846+000
L(-4,108) = 1.8002722251+000	L(108, 4) = 1.8002722251+000	L(-4, 4) = 1.8600990376+000
L(5, 5) = 1.5729563607+000	L(-5, 5) = 1.5729563607+000	L(5, 5) = 1.8639183761+000
L(109, 5) = 1.7986762149+000	L(109, 5) = 1.7986762149+000	L(109, 109) = 2.2352361261+000
L(-5,109) = 1.7986762149+000	L(109, 5) = 1.7986762149+000	L(-5, 5) = 1.8639183761+000
L(6, 6) = 1.5785891898+000	L(-6, 6) = 1.5785891898+000	L(6, 6) = 1.8686743510+000
L(110, 6) = 1.7966717521+000	L(110, 6) = 1.7966717521+000	L(110, 110) = 2.2280293847+000
L(-6,110) = 1.7966717521+000	L(110, 6) = 1.7966717521+000	L(-6, 6) = 1.8686743510+000

LONG DIAGONALS

L(1, 1) = 1.6181263264+000	L(-1, 1) = 1.6181263264+000	L(1, 1) = 1.9021915803+000
L(105, 1) = 1.7819773854+000	L(105, 1) = 1.7819773854+000	L(105, 105) = 2.1754434022+000
L(-1,105) = 1.7819773854+000	L(105, 1) = 1.7819773854+000	L(-1, 1) = 1.9021915803+000
L(2, 2) = 1.5746028711+000	L(-2, 2) = 1.5746028711+000	L(2, 2) = 1.8653080716+000
L(106, 2) = 1.7980924807+000	L(106, 2) = 1.7980924807+000	L(106, 106) = 2.2331365692+000
L(-2,106) = 1.7980924807+000	L(106, 2) = 1.7980924807+000	L(-2, 2) = 1.8653080716+000
L(3, 3) = 1.5674109943+000	L(-3, 3) = 1.5674109943+000	L(3, 3) = 1.8592410347+000
L(107, 3) = 1.8006291065+000	L(107, 3) = 1.8006291065+000	L(107, 107) = 2.2422651790+000
L(-3,107) = 1.8006291065+000	L(107, 3) = 1.8006291065+000	L(-3, 3) = 1.8592410347+000
L(4, 4) = 1.5684286499+000	L(-4, 4) = 1.5684286499+000	L(4, 4) = 1.8600990376+000
L(108, 4) = 1.8002722251+000	L(108, 4) = 1.8002722251+000	L(108, 108) = 2.2409800846+000
L(-4,108) = 1.8002722251+000	L(108, 4) = 1.8002722251+000	L(-4, 4) = 1.8600990376+000
L(5, 5) = 1.5729563607+000	L(-5, 5) = 1.5729563607+000	L(5, 5) = 1.8639183761+000
L(109, 5) = 1.7986762149+000	L(109, 5) = 1.7986762149+000	L(109, 109) = 2.2352361261+000
L(-5,109) = 1.7986762149+000	L(109, 5) = 1.7986762149+000	L(-5, 5) = 1.8639183761+000

SHORT DIAGONALS

| L(7, 7) = 1.5825165946+000 |
|-------------------------------|-------------------------------|-------------------------------|-------------------------------|
| L(111, 7) = 1.7952615730+000 |
L(7,111) = 1.7952615730+000	L(7,111) = 1.7952615730+000	L(7,111) = 1.7952615730+000	L(7, 7) = 1.8719932618+000
L(112,112) = 1.7612895053+000	L(8, 8) = 1.7012509060+000	L(8, 8) = 1.7012509060+000	L(8,112) = 1.9990986385+000
L(8, 8) = 1.7012509060+000	L(112,112) = 1.7612895053+000	L(112,112) = 1.7612895053+000	L(8,112) = 1.9990986385+000
L(112,112) = 1.7612895053+000	L(8, 8) = 1.7012509060+000	L(8, 8) = 1.7012509060+000	L(112, 8) = 1.9990986385+000
L(10, 10) = 1.9019073767+000	L(11, 11) = 1.8281209615+000	L(11, 11) = 1.8281209615+000	L(11, 10) = 2.1158725773+000
L(9, 11) = 1.5962700058+000	L(12, 10) = 1.6964813095+000	L(12, 10) = 1.6964813095+000	L(9, 12) = 1.7142304068+000
L(10, 12) = 1.6964813095+000	L(9, 11) = 1.5962700058+000	L(9, 11) = 1.5962700058+000	L(10, 11) = 2.1158725773+000
L(14, 14) = 1.5801787586+000	L(15, 15) = 1.5998847845+000	L(15, 15) = 1.5998847845+000	L(15, 14) = 1.8783247730+000
L(13, 15) = 1.8004147650+000	L(16, 14) = 1.7844695743+000	L(16, 14) = 1.7844695743+000	L(13, 16) = 2.2130199185+000
L(14, 16) = 1.7844695743+000	L(13, 15) = 1.8004147650+000	L(13, 15) = 1.8004147650+000	L(14, 15) = 1.8783247730+000
L(8, 18) = 1.8857050313+000	L(19, 19) = 1.8421030112+000	L(19, 19) = 1.8421030112+000	L(19, 18) = 2.1151035238+000
L(17, 19) = 1.6202320902+000	L(20, 18) = 1.6789449535+000	L(20, 18) = 1.6789449535+000	L(20, 17) = 1.7224134580+000
L(18, 20) = 1.6789449535+000	L(17, 19) = 1.6202320902+000	L(17, 19) = 1.6202320902+000	L(18, 19) = 2.1151035238+000
L(22, 22) = 1.8674731938+000	L(23, 23) = 1.8289787017+000	L(23, 23) = 1.8289787017+000	L(23, 22) = 2.1013254621+000
L(21, 23) = 1.6370233217+000	L(24, 22) = 1.6864972094+000	L(24, 22) = 1.6864972094+000	L(21, 24) = 1.7623855703+000
L(22, 24) = 1.6864972094+000	L(21, 23) = 1.6370233217+000	L(21, 23) = 1.6370233217+000	L(22, 23) = 2.1013254621+000
L(26, 26) = 1.6178287392+000	L(27, 27) = 1.6699502989+000	L(27, 27) = 1.6699502989+000	L(27, 26) = 1.9239785827+000
L(25, 27) = 1.7937712972+000	L(28, 26) = 1.7483367231+000	L(28, 26) = 1.7483367231+000	L(25, 28) = 2.1378744893+000
L(26, 28) = 1.7483367231+000	L(25, 27) = 1.7937712972+000	L(25, 27) = 1.7937712972+000	L(26, 27) = 1.9239785827+000
L(30, 30) = 1.8307758474+000	L(31, 31) = 1.7832869170+000	L(31, 31) = 1.7832869170+000	L(31, 30) = 2.0651388856+000
L(29, 31) = 1.6617605205+000	L(32, 30) = 1.7168472408+000	L(32, 30) = 1.7168472408+000	L(29, 32) = 1.8550292440+000
L(30, 32) = 1.7168472408+000	L(29, 31) = 1.6617605205+000	L(29, 31) = 1.6617605205+000	L(30, 31) = 2.0651388856+000
L(34, 38) = 1.8796312136+000	L(35, 37) = 1.8345194473+000	L(35, 37) = 1.8345194473+000	L(35, 38) = 2.1885697020+000
L(33, 35) = 1.6673589800+000	L(36, 38) = 1.7350380059+000	L(36, 38) = 1.7350380059+000	L(33, 36) = 1.7379062276+000
L(34, 36) = 1.6325304796+000	L(33, 37) = 1.5702672582+000	L(33, 37) = 1.5702672582+000	L(34, 37) = 2.0264711480+000
L(40, 44) = 1.8582463n37+000	L(41, 43) = 1.8300178870+000	L(41, 43) = 1.8300178870+000	L(41, 44) = 2.2139306134+000
L(39, 41) = 1.7199834164+000	L(42, 44) = 1.7532489305+000	L(42, 44) = 1.7532489305+000	L(45, 48) = 1.7972963831+000
L(40, 42) = 1.615040798+000	L(39, 43) = 1.5667260783+000	L(39, 43) = 1.5667260783+000	L(46, 49) = 1.9436482955+000
L(46, 50) = 1.8416840720+000	L(47, 49) = 1.8214800583+000	L(47, 49) = 1.8214800583+000	L(53, 56) = 2.2190757492+000
L(45, 47) = 1.7439205431+000	L(48, 50) = 1.7666254963+000	L(48, 50) = 1.7666254963+000	L(51, 54) = 1.8248524394+000
L(46, 48) = 1.5958773555+000	L(45, 49) = 1.5690580454+000	L(45, 49) = 1.5690580454+000	L(52, 55) = 1.9214770375+000

LONG DIAGONALS

L(58, 62) = 1.8135638385+000	L(59, 61) = 1.7976807544+000	L(59, 62) = 2.2150112626+000
L(57, 59) = 1.7718721465+000	L(60, 62) = 1.7883771796+000	L(57, 60) = 1.8523672194+000
L(58, 60) = 1.5983634057+000	L(57, 61) = 1.5783015972+000	L(58, 61) = 1.9010876843+000
L(64, 68) = 1.8731931978+000	L(65, 67) = 1.8367053647+000	L(65, 68) = 2.1591627634+000
L(63, 65) = 1.6663729607+000	L(66, 68) = 1.71269663586+000	L(63, 66) = 1.7452417121+000
L(64, 66) = 1.6472984826+000	L(63, 67) = 1.5980619238+000	L(64, 67) = 2.0540251673+000
L(70, 74) = 1.8554378583+000	L(71, 73) = 1.8235501679+000	L(71, 74) = 2.1783026417+000
L(69, 71) = 1.7015443615+000	L(72, 74) = 1.7394704393+000	L(69, 72) = 1.781710696+000
L(70, 72) = 1.6330472561+000	L(69, 73) = 1.5910370971+000	L(70, 73) = 2.0054814184+000
L(76, 80) = 1.8349658082+000	L(77, 79) = 1.8045063580+000	L(77, 80) = 2.1843010052+000
L(75, 77) = 1.7299564409+000	L(78, 80) = 1.7638180053+000	L(75, 78) = 1.8252218767+000
L(76, 78) = 1.6292508809+000	L(75, 79) = 1.5905081893+000	L(76, 79) = 1.9624548374+000
L(82, 86) = 1.8122071589+000	L(83, 85) = 1.7812261353+000	L(83, 86) = 2.1850000610+000
L(81, 83) = 1.7559626551+000	L(84, 86) = 1.7880706458+000	L(81, 84) = 1.8722476185+000
L(82, 84) = 1.6295039418+000	L(81, 85) = 1.5915781538+000	L(82, 85) = 1.9187635410+000
L(88, 92) = 1.8502996457+000	L(89, 91) = 1.8101626475+000	L(89, 92) = 2.1255863734+000
L(87, 89) = 1.6751749261+000	L(90, 92) = 1.7231651510+000	L(87, 90) = 1.8041635797+000
L(88, 90) = 1.6739521134+000	L(87, 91) = 1.6235897459+000	L(88, 91) = 2.0446431830+000
L(94, 98) = 1.8250676391+000	L(95, 97) = 1.7822701844+000	L(95, 98) = 2.1354897162+000
L(93, 95) = 1.7120476215+000	L(96, 98) = 1.7590094967+000	L(93, 96) = 1.8608178650+000
L(94, 96) = 1.6703050835+000	L(93, 97) = 1.6188225314+000	L(94, 97) = 1.9862521310+000
L(100,104) = 1.7995418383+000	L(101,103) = 1.7459414098+000	L(101,104) = 2.0730147345+000
L(99,101) = 1.7035631316+000	L(102,104) = 1.7599998952+000	L(99,102) = 1.9233116791+000
L(100,102) = 1.7159350212+000	L(99,103) = 1.6566850514+000	L(100,103) = 1.9965955015+000

DIHEDRAL ANGLES -- EXPRESSED IN DEG

	ORDERED TRIPLET	DIHD ANGLE	ORDERED TRIPLET	DIHD ANGLE	ORDERED TRIPLET	DIHD ANGLE	ORDERED TRIPLET	DIHD ANGLE	ORDERED TRIPLET	DIHD ANGLE	ORDERED TRIPLET	DIHD ANGLE	ORDERED TRIPLET	DIHD ANGLE
113	1 1	178.9585946	1 113	1	178.7757986	1	10	2	177.5821809					
10	2 1	178.0208701	10 1	2	177.9661424	2	38	3	176.4677130					
38	3 2	177.1541031	38 2	3	177.1410343	3	44	4	175.5278866					
44	4 3	176.3999123	44 3	4	176.4022512	4	50	5	174.7676562					
50	5 4	175.7725542	50 4	5	175.7847448	5	56	6	174.2129083					
56	6 5	175.2994231	56 5	6	175.3162335	6	62	7	173.8803418					
62	7 6	175.0080580	62 6	7	175.0204780	7	13	7	173.7704884					
13	7 7	174.9145912	12 34	34	176.9246448	34	12	34	177.5064086					
36	18 40	175.9475036	18 40	36	176.2558635	18	36	40	176.8838289					
42	68 46	175.1217782	68 46	42	175.6666805	68	42	46	176.2537560					
48	74 52	174.4836186	74 52	48	175.2139556	74	48	52	175.6982756					
54	80 58	174.0583845	80 58	54	174.9387899	80	54	58	175.2627051					
60	86 15	173.8498626	86 15	60	174.8610571	86	60	15	174.9757351					
20	64 64	175.4512471	64 20	64	176.4602179	66	22	70	174.7936617					
22	70 66	175.0502613	22 66	70	175.9781240	72	92	76	174.3262906					
92	76 72	174.8062692	92 72	76	175.5209623	78	98	82	174.0595009					
98	82 78	174.7493853	98 78	82	175.1404868	84	25	84	173.9750598					
25	84 84	174.8716263	24 88	88	174.5980050	88	24	88	175.6340152					
90	30 94	174.3264462	30 94	90	174.5705885	30	90	94	175.2794644					
96	104 27	174.2096460	104 27	96	174.7044838	104	96	27	174.9568113					
32	100 100	174.4749438	100 32	100	175.0033578	102	112	102	174.4545912					
112	102 102	174.7210067												

Execution time--27 sec

INITIAL GUESS

```

AG(1) THRU AG(120) = 1.00-001
AG(121) = -2.00-001
AG(122) = -2.00-001
AG(123) = -2.00-001
AG(124) = -2.00-001
AG(125) = -2.00-001
AG(126) = -2.00-001
AG(127) = -2.00-001
AG(128) = -2.00-001
AG(129) = -1.20+001

```

FACE ANGLES -- EXPRESSED AS EXCESS OVER 120 DEG

6 ITERATIONS REQUIRED

INDEX	EXCESS (DEG)	INDEX	EXCESS (DEG)	INDEX	EXCESS (DEG)
1	5.996405+000	44	-1.387561+000	87	-1.145509+001
2	8.072226+000	45	-1.337276+001	88	1.009562+001
3	8.421521+000	46	1.258345+001	89	2.934728+000
4	8.405430+000	47	2.391700+000	90	-1.484530+001
5	8.232563+000	48	-1.691560+001	91	1.112498+001
6	7.9886262+000	49	1.431458+001	92	2.026755+000
7	7.7663355+000	50	1.3233897+000	93	-1.146934+001
8	7.677427+000	51	-1.434721+001	94	7.505988+000
9	7.396955+001	52	1.153856+001	95	5.529300+000
10	-1.409563+001	53	3.931309+000	96	-1.471768+001
11	-3.973229+000	54	-1.676113+001	97	1.631834+001
12	1.216817+001	55	1.239594+001	98	-6.629960+000
13	2.116674+001	56	3.073175+000	99	-7.111248+000
14	-1.182149+001	57	-1.447247+001	100	1.106242+001
15	-5.955316+000	58	1.023435+001	101	-1.882410+000
16	1.436688+001	59	5.142201+000	102	-1.175714+001
17	-1.464138+001	60	-1.637319+001	103	-1.317332+001
18	8.280406+000	61	1.066473+001	104	-3.018582+000
19	4.748100+000	62	4.441998+000	105	-7.689546+000

Execution time--21 sec

FACE ANGLES -- EXPRESSED AS EXCESS COVER 120 DEG

6 ITERATIONS REQUIRED

INDEX	EXCESS(DEG)	INDEX	EXCESS(DEG)	INDEX	EXCESS(DEG)
20	-1.141563+001	63	-1.423346+001	106	8.049607+000
21	1.858072+001	64	8.756609+000	107	1.787641+000
22	-1.031973+001	65	6.300706+000	108	-1.230244+001
23	-5.522359+000	66	-1.593058+001	109	9.503574+000
24	1.310345+001	67	9.048123+000	110	6.683287-001
25	1.406236+001	68	5.774526+000	111	-7.789869+000
26	-8.218961+000	69	-1.399493+001	112	4.446222+000
27	-2.968722+000	70	7.224363+000	113	5.616379+000
28	8.313009+000	71	7.582334+000	114	-1.244463+001
29	-9.083413+000	72	-1.563442+001	115	1.048539+001
30	6.116896+000	73	1.923693+001	116	-4.249136+000
31	8.211365-002	74	-7.235800+000	117	-3.284575+000
32	-3.314606+000	75	-9.348644+000	118	4.459538+000
33	6.391566+000	76	1.380230+001	119	1.493696+000
34	-4.731783+000	77	-2.437168+000	120	-8.904912+000
35	1.567819+000	78	-1.401762+001	121	-1.199281+001
36	-6.363925-002	79	1.666137+001	122	-1.614445+001
37	2.007845+001	80	-3.616265+000	123	-1.684304+001
38	-6.108565+000	81	-1.093848+001	124	-1.681086+001
39	-1.062420+001	82	1.226679+001	125	-1.646513+001
40	1.313135+001	83	3.805488-001	126	-1.597252+001
41	7.202969-002	84	-1.475397+001	127	-1.553267+001
42	-1.654906+001	85	1.393989+001	128	-1.535485+001
43	1.670078+001	86	-6.698522-001	129	-1.200000+001

HEXAGON DIAGONALS

SHORT DIAGONALS

LONG DIAGONALS

$L(1, 1) = 1.618108$	$L(1, 1) = 1.618108$	$L(1, 1) = 1.618108$
$L(121, 1) = 1.781985$	$L(121, 1) = 1.781985$	$L(121, 1) = 1.781985$
$L(-1, 121) = 1.781985$	$L(-1, 121) = 1.781985$	$L(-1, 1) = 1.902176$
$L(2, 2) = 1.574468$	$L(2, 2) = 1.574468$	$L(2, 2) = 1.865194$
$L(122, 2) = 1.798140$	$L(122, 2) = 1.798140$	$L(122, 2) = 2.233309$
$L(-2, 122) = 1.798140$	$L(-2, 122) = 1.798140$	$L(-2, 2) = 1.865194$
$L(3, 3) = 1.566920$	$L(3, 3) = 1.566920$	$L(3, 3) = 1.858827$
$L(123, 3) = 1.800801$	$L(123, 3) = 1.800801$	$L(123, 3) = 2.242884$
$L(-3, 123) = 1.800801$	$L(-3, 123) = 1.800801$	$L(-3, 3) = 1.858827$
$L(4, 4) = 1.567269$	$L(4, 4) = 1.567269$	$L(4, 4) = 1.859121$
$L(124, 4) = 1.800679$	$L(124, 4) = 1.800679$	$L(124, 4) = 2.242444$
$L(-4, 124) = 1.800679$	$L(-4, 124) = 1.800679$	$L(-4, 4) = 1.859121$
$L(5, 5) = 1.571011$	$L(5, 5) = 1.571011$	$L(5, 5) = 1.862277$
$L(125, 5) = 1.799364$	$L(125, 5) = 1.799364$	$L(125, 5) = 2.237710$
$L(-5, 125) = 1.799364$	$L(-5, 125) = 1.799364$	$L(-5, 5) = 1.862277$
$L(6, 6) = 1.576317$	$L(6, 6) = 1.576317$	$L(6, 6) = 1.866755$
$L(126, 6) = 1.797483$	$L(126, 6) = 1.797483$	$L(126, 6) = 2.230945$
$L(-6, 126) = 1.797483$	$L(-6, 126) = 1.797483$	$L(-6, 6) = 1.866755$
$L(7, 7) = 1.581030$	$L(7, 7) = 1.581030$	$L(7, 7) = 1.870737$
$L(127, 7) = 1.795797$	$L(127, 7) = 1.795797$	$L(127, 7) = 2.224885$
$L(-7, 127) = 1.795797$	$L(-7, 127) = 1.795797$	$L(-7, 7) = 1.870737$
$L(8, 8) = 1.582929$	$L(8, 8) = 1.582929$	$L(8, 8) = 1.872342$
$L(128, 8) = 1.795113$	$L(128, 8) = 1.795113$	$L(128, 8) = 2.222431$
$L(-8, 128) = 1.795113$	$L(-8, 128) = 1.795113$	$L(-8, 8) = 1.872342$
$L(10, 10) = 1.901949$	$L(10, 10) = 1.901949$	$L(10, 10) = 2.115963$
$L(9, 11) = 1.596266$	$L(9, 11) = 1.596266$	$L(9, 11) = 2.115963$
$L(10, 12) = 1.696344$	$L(10, 12) = 1.696344$	$L(10, 11) = 2.115963$
$L(14, 14) = 1.886368$	$L(14, 14) = 1.886368$	$L(14, 14) = 2.116016$
$L(13, 15) = 1.619863$	$L(13, 15) = 1.619863$	$L(13, 15) = 2.116016$
$L(14, 16) = 1.677766$	$L(14, 16) = 1.677766$	
		$L(14, 15) = 1.619863$

HEXAGON DIAGONALS

HEXAGON DIAGONALS		SHORT DIAGONALS		LONG DIAGONALS	
L(18, 18) = 1.590509	L(19, 19) = 1.624008	L(19, 18) = 1.799728	L(20, 19) = 1.771987	L(19, 18) = 1.897787	L(19, 18) = 1.892881
L(17, 19) = 1.799728	L(20, 19) = 1.771987	L(17, 19) = 1.799728	L(20, 19) = 1.771987	L(17, 20) = 1.897787	L(18, 19) = 1.892881
L(18, 20) = 1.771987	L(17, 19) = 1.799728	L(21, 23) = 1.834839	L(22, 23) = 1.834839	L(21, 22) = 1.751392	L(21, 22) = 1.751392
L(22, 22) = 1.870769	L(23, 23) = 1.834839	L(24, 22) = 1.681867	L(21, 23) = 1.635097	L(22, 23) = 1.635097	L(22, 23) = 2.105365
L(21, 23) = 1.635097	L(24, 22) = 1.681867	L(21, 23) = 1.635097	L(22, 23) = 1.635097	L(21, 22) = 1.751392	L(21, 22) = 2.105365
L(22, 24) = 1.681867	L(21, 23) = 1.635097	L(27, 27) = 1.799976	L(26, 27) = 1.799976	L(27, 26) = 1.825930	L(26, 27) = 2.077147
L(26, 26) = 1.841435	L(28, 26) = 1.705565	L(28, 26) = 1.705565	L(25, 25) = 1.655935	L(25, 26) = 1.825930	L(26, 27) = 2.077147
L(25, 27) = 1.655935	L(25, 27) = 1.655935	L(.25, .27) = 1.655935.	L(.25, .27) = 1.655935.	L(.26, .27) = 1.655935.	L(.26, .27) = 2.077147
L(26, 28) = 1.705565	L(.25, .27) = 1.655935.	L(31, 31) = 1.702405	L(31, 31) = 1.702405	L(31, 30) = 1.950535	L(31, 30) = 1.950535
L(30, 30) = 1.647427	L(31, 31) = 1.702405	L(32, 30) = 1.732767	L(29, 31) = 1.732767	L(29, 32) = 1.91466	L(29, 32) = 2.091466
L(29, 31) = 1.782938	L(32, 30) = 1.732767	L(29, 31) = 1.782938	L(30, 31) = 1.782938	L(30, 31) = 1.950535	L(30, 31) = 1.950535
L(30, 32) = 1.732767	L(29, 31) = 1.782938	L(35, 35) = 1.731495	L(35, 35) = 1.731495	L(35, 34) = 2.022598	L(35, 34) = 2.022598
L(34, 34) = 1.785105	L(35, 35) = 1.731495	L(40, 42) = 1.745570	L(40, 42) = 1.745570	L(33, 36) = 1.951065	L(33, 36) = 1.951065
L(33, 35) = 1.689294	L(36, 34) = 1.745570	L(36, 34) = 1.745570	L(37, 41) = 1.570104	L(37, 40) = 1.737011	L(37, 40) = 1.737011
L(34, 36) = 1.745570	L(35, 35) = 1.689294	L(33, 35) = 1.689294	L(34, 35) = 1.689294	L(34, 35) = 2.022598	L(34, 35) = 2.022598
L(38, 42) = 1.879853	L(39, 41) = 1.835033	L(40, 42) = 1.732679	L(40, 42) = 1.732679	L(39, 42) = 2.188966	L(39, 42) = 2.188966
L(37, 39) = 1.676368	L(40, 42) = 1.732679	L(41, 43) = 1.570104	L(41, 43) = 1.570104	L(37, 40) = 1.737011	L(37, 40) = 1.737011
L(38, 40) = 1.632031	L(37, 41) = 1.570104	L(45, 47) = 1.831209	L(45, 47) = 1.831209	L(45, 48) = 2.215008	L(45, 48) = 2.215008
L(44, 48) = 1.858915	L(45, 47) = 1.831209	L(46, 48) = 1.752544	L(46, 48) = 1.752544	L(43, 46) = 1.766852	L(43, 46) = 1.766852
L(43, 45) = 1.719815	L(47, 43) = 1.566133	L(43, 47) = 1.566133	L(44, 47) = 1.566133	L(44, 47) = 1.975310	L(44, 47) = 1.975310
L(44, 46) = 1.603835	L(47, 43) = 1.566133	L(51, 53) = 1.823800	L(51, 53) = 1.823800	L(51, 54) = 2.222806	L(51, 54) = 2.222806
L(50, 54) = 1.843148	L(51, 53) = 1.823800	L(52, 54) = 1.765332	L(49, 53) = 1.765332	L(49, 52) = 1.792979	L(49, 52) = 1.792979
L(49, 51) = 1.743488	L(52, 54) = 1.765332	L(55, 53) = 1.567808	L(50, 53) = 1.567808	L(50, 53) = 1.944786	L(50, 53) = 1.944786
L(50, 52) = 1.593617	L(49, 53) = 1.567808	L(57, 59) = 1.814340	L(57, 59) = 1.814340	L(57, 60) = 2.222187	L(57, 60) = 2.222187
L(56, 60) = 1.829891	L(57, 59) = 1.814340	L(58, 60) = 1.775166	L(55, 58) = 1.784390	L(55, 58) = 1.817750	L(55, 58) = 1.817750
L(55, 57) = 1.758243	L(58, 60) = 1.775166	L(59, 61) = 1.576767	L(56, 59) = 1.576767	L(61, 64) = 1.842441	L(61, 64) = 1.842441
L(56, 58) = 1.592295	L(59, 61) = 1.576767	L(63, 65) = 1.803358	L(63, 65) = 1.803358	L(62, 65) = 1.906493	L(62, 65) = 1.906493
L(62, 66) = 1.817488	L(63, 65) = 1.803358	L(64, 66) = 1.784390	L(64, 66) = 1.784390	L(63, 66) = 2.18190	L(63, 66) = 2.18190
L(61, 63) = 1.769504	L(64, 66) = 1.784390	L(65, 67) = 1.576767	L(65, 67) = 1.576767	L(67, 70) = 1.866350	L(67, 70) = 1.866350
L(62, 64) = 1.594816	L(65, 67) = 1.576767	L(69, 71) = 1.791613	L(69, 71) = 1.791613	L(68, 71) = 1.887881	L(68, 71) = 1.887881
L(68, 72) = 1.805532	L(69, 71) = 1.791613	L(70, 72) = 1.794381	L(70, 72) = 1.794381	L(68, 71) = 1.887881	L(68, 71) = 1.887881
L(67, 69) = 1.780223	L(70, 72) = 1.794381	L(67, 71) = 1.579942	L(67, 71) = 1.579942	L(68, 71) = 1.887881	L(68, 71) = 1.887881
L(68, 70) = 1.597324	L(67, 71) = 1.579942				

L(74, 78) = 1.874789	L(75, 77) = 1.839659	L(75, 78) = 2.161184
L(73, 75) = 1.665497	L(76, 78) = 1.710392	L(73, 76) = 1.739673
L(74, 76) = 1.644798	L(73, 77) = 1.597086	L(74, 77) = 2.056021
L(80, 84) = 1.858661	L(81, 83) = 1.828980	L(81, 84) = 2.181776
L(79, 81) = 1.699636	L(82, 84) = 1.735362	L(79, 82) = 1.771651
L(80, 82) = 1.628854	L(79, 83) = 1.589317	L(80, 83) = 2.009665
L(86, 90) = 1.840600	L(87, 89) = 1.813320	L(87, 90) = 2.189040
L(85, 87) = 1.726176	L(88, 90) = 1.757090	L(85, 88) = 1.809497
L(86, 88) = 1.623606	L(85, 89) = 1.588349	L(86, 89) = 1.970638
L(92, 96) = 1.820826	L(93, 95) = 1.793792	L(93, 96) = 2.190060
L(91, 93) = 1.749466	L(94, 96) = 1.778268	L(91, 94) = 1.850850
L(92, 94) = 1.623461	L(91, 95) = 1.589701	L(92, 95) = 1.932870
L(98,102) = 1.856442	L(99,101) = 1.820374	L(99,102) = 2.132009
L(97, 99) = 1.671328	L(100,102) = 1.715391	L(97,100) = 1.785614
L(98,100) = 1.666699	L(97,101) = 1.620522	L(98,101) = 2.052650
L(104,105) = 1.835324	L(105,107) = 1.797967	L(105,108) = 2.144028
L(103,105) = 1.705111	L(106,108) = 1.747440	L(103,106) = 1.833641
L(104,106) = 1.661099	L(103,107) = 1.614926	L(104,107) = 2.000710
L(110,114) = 1.808937	L(111,113) = 1.769538	L(111,114) = 2.148020
L(109,111) = 1.737854	L(112,114) = 1.778963	L(109,112) = 1.889214
L(110,112) = 1.660123	L(109,113) = 1.613460	L(110,113) = 1.946267
L(116,120) = 1.816180	L(117,119) = 1.769646	L(117,120) = 2.088419
L(115,117) = 1.693788	L(118,120) = 1.744938	L(115,118) = 1.884033
L(116,118) = 1.702680	L(115,119) = 1.646192	L(116,119) = 2.016555

DIHEDRAL ANGLES -- EXPRESSED IN DEG

ORDERED TRIPLET	DIHD ANGLE								
129 1 1	179.06923	1 129	178.90585	1	10 2	177.83852			
10 2 1	178.23082	10 1	178.18177	2	42 3	176.83842			
42 3 2	177.45362	42 2	177.44135	3	48 4	175.98490			
48 4 3	176.77070	48 3	176.77142	4	54 5	175.27546			
54 5 4	176.18904	54 4	176.19812	5	60 6	174.72952			
60 6 5	175.72777	60 5	175.74218	6	66 7	174.36385			
66 7 6	175.40925	66 6	175.42296	7	72 8	174.18263			
72 8 7	175.24987	72 7	175.25558	12	38 38	177.24855			
38 12 78	177.76987	40 14	176.36585	14	44 40	176.64239			
14 40 44	177.20928	46 78	175.60479	78	50 46	176.09527			
78 46 50	176.63508	52 84	174.99219	84	56 52	175.65187			
84 52 56	176.11684	58 90	174.55098	90	62 58	175.34619			
90 58 62	175.69273	64 96	174.28995	96	68 64	175.19981			
96 64 68	175.38666	70 17	174.20443	17	70 70	175.21683			
16 74 74	175.90271	74 16	176.82459	76	22 80	175.27627			
22 80 76	175.50627	22 76	176.38093	82	102 86	174.80091			
102 86 82	175.22853	102 82	175.95114	88	108 92	174.49380			
108 92 88	175.10151	108 88	175.58110	94	114 19	174.34779			
114 19 94	175.13094	114 94	175.29948	24	98 98	175.04435			
98 24 98	176.06022	100 26	174.73344	26	104 100	174.94643			
26 100 104	175.72581	106 120	174.56712	120	110 106	174.99247			
120 106 110	175.41641	112 129	174.51712	129	112 112	175.16116			
28 116 116	174.79866	116 28	175.47037	118	34 31	174.74518			
34 31 118	174.97246	34 118	175.20998	36	36 36	174.97628			

INITIAL GUESS

```

AG(1) THRU AG(136) = 1.00+001
AG(137) = -2.00+001
AG(138) = -2.00+001
AG(139) = -2.00+001
AG(140) = -2.00+001
AG(141) = -2.00+001
AG(142) = -2.00+001
AG(143) = -2.00+001
AG(144) = -2.00+001
AG(145) = -1.20+001

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FACE ANGLES -- EXPRESSED AS EXCESS OVER 120 DFG

6 ITERATIONS REQUIRED

INDEX	EXCESS(DFG)	INDEX	EXCESS(DFG)	INDEX	EXCESS(DFG)
1	5.997069+000	50	1.271051+001	99	4.605988+000
2	8.076966+000	51	2.330229+000	100	-1.490017+001
3	8.438822+000	52	-1.695843+001	101	9.490155+000
4	8.447397+000	53	1.447904+001	102	3.860563+000
5	8.307276+000	54	1.290003+000	103	-1.189454+001
6	8.084566+000	55	=1.451393+001	104	6.322166+000
7	7.853000+000	56	1.178108+001	105	6.982280+000
8	7.703458+000	57	3.819584+001	106	-1.476062+001
9	2.397534+001	58	=1.685577+001	107	1.702370+001
10	-1.404592+001	59	1.268103+001	108	-6.927983+000
11	-3.984457+000	60	2.989778+000	109	-7.684770+000
12	1.218542+001	61	-1.472789+001	110	1.210994+001
13	-1.557723+001	62	1.063681+001	111	-2.525770+000
14	8.228266+000	63	4.940467+000	112	-1.199512+001
15	6.625565+000	64	=1.652019+001	113	1.428749+001
16	-1.413043+001	65	1.108967+001	114	-3.583830+000
17	2.127298+001	66	4.250332+000	115	-8.447939+000
18	-1.184905+001	67	-1.453832+001	116	9.609847+000
19	-8.048034+000	68	4.340158+000	117	7.485685-001

Execution time--25 sec

FACE ANGLES -- EXPRESSED AS EXCESS COVER 120 DFG

6 ITERATIONS REQUIRED

INDEX	EXCESS(DFG)	INDEX	EXCESS(DEG)	INDEX	EXCESS(DEG)
20	1.452119*001	69	5.946405*000	118	*1.261413*001
21	1.897946*001	70	=1.608824*001	119	1.107743*001
22	-1.046630*001	71	9.602025*000	120	*2.749377*001
23	-5.886127*000	72	5.400172*000	121	*8.637989*000
24	1.372539*001	73	=1.425740*001	122	6.505818*000
25	-1.281423*001	74	7.965680*000	123	4.119843*000
26	7.612978*000	75	7.011237*000	124	*1.279017*001
27	3.117868*000	76	=1.572172*001	125	1.222707*001
28	-8.647461*000	77	*1.943419*001	126	*5.065513*000
29	1.523405*001	78	=7.303384*000	127	*4.477663*000
30	-2.674596*000	79	=9.537684*000	128	6.758331*000
31	-3.928105*000	80	*1.412197*001	129	5.416411*002
32	9.971357*000	81	=2.624368*000	130	*9.496387*000
33	2.799838*000	82	=1.409072*001	131	8.426467*000
34	-5.753119*000	83	1.703699*001	132	*1.171597*000
35	-7.473985*002	84	=3.765901*000	133	*4.593645*000
36	2.8558881*000	85	=1.125628*001	134	2.886960*000
37	-5.942491*000	86	1.283635*001	135	4.243659*000
38	4.509862*000	87	3.727785*002	136	*9.791845*000
39	-1.512167*000	88	=1.488844*001	137	*1.199418*001
40	-5.289981*002	89	1.456366*001	138	*1.615393*001
41	2.010659*001	90	=9.751155*001	139	*1.687764*001
42	-6.112558*000	91	=1.190081*001	140	*1.689479*001
43	-1.066153*001	92	1.099454*001	141	*1.661455*001
44	1.318697*001	93	2.346711*000	142	*1.616913*001
45	4.119104*002	94	=1.502899*001	143	*1.570600*001
46	-1.656066*001	95	1.204381*001	144	*1.540692*001
47	1.677936*001	96	1.466408*000	145	*1.200000*001
48	-1.401107*000	97	*1.198000*001		
49	-1.346056*001	98	8.763975*000		

HEXAGON DIAGONALS

SHORT DIAGONALS

LONG DIAGONALS

$L(1, 1) = 1.618094$	$L(1, 1) = 1.618094$	$L(1, 1) = 1.618094$
$L(137, 1) = 1.781990$	$L(137, 1) = 1.781990$	$L(137, 1) = 1.781990$
$L(-1, 137) = 1.781990$	$L(-1, 137) = 1.781990$	$L(-1, 1) = 1.902164$
$L(2, 2) = 1.574366$	$L(2, 2) = 1.574366$	$L(2, 2) = 1.865108$
$L(138, 2) = 1.798177$	$L(138, 2) = 1.798177$	$L(138, 138) = 2.233439$
$L(-2, 138) = 1.798177$	$L(-2, 138) = 1.798177$	$L(-2, 2) = 1.865108$
$L(3, 3) = 1.566545$	$L(3, 3) = 1.566545$	$L(3, 3) = 1.858511$
$L(139, 3) = 1.800932$	$L(139, 3) = 1.800932$	$L(139, 139) = 2.243357$
$L(-3, 139) = 1.800932$	$L(-3, 139) = 1.800932$	$L(-3, 3) = 1.858511$
$L(4, 4) = 1.566359$	$L(4, 4) = 1.566359$	$L(4, 4) = 1.858354$
$L(140, 4) = 1.800997$	$L(140, 4) = 1.800997$	$L(140, 140) = 2.243592$
$L(-4, 140) = 1.800997$	$L(-4, 140) = 1.800997$	$L(-4, 4) = 1.858354$
$L(5, 5) = 1.569395$	$L(5, 5) = 1.569395$	$L(5, 5) = 1.860914$
$L(141, 5) = 1.794953$	$L(141, 5) = 1.794953$	$L(141, 141) = 2.239757$
$L(-5, 141) = 1.799933$	$L(-5, 141) = 1.799933$	$L(-5, 5) = 1.860914$
$L(6, 6) = 1.574202$	$L(6, 6) = 1.574202$	$L(6, 6) = 1.864970$
$L(142, 6) = 1.798235$	$L(142, 6) = 1.798235$	$L(142, 142) = 2.233648$
$L(-6, 142) = 1.798235$	$L(-6, 142) = 1.798235$	$L(-6, 6) = 1.864970$
$L(7, 7) = 1.579175$	$L(7, 7) = 1.579175$	$L(7, 7) = 1.869170$
$L(143, 7) = 1.796462$	$L(143, 7) = 1.796462$	$L(143, 143) = 2.227275$
$L(-7, 143) = 1.796462$	$L(-7, 143) = 1.796462$	$L(-7, 7) = 1.869170$
$L(8, 8) = 1.582373$	$L(8, 8) = 1.582373$	$L(8, 8) = 1.871872$
$L(144, 8) = 1.795313$	$L(144, 8) = 1.795313$	$L(144, 144) = 2.223150$
$L(-8, 144) = 1.795313$	$L(-8, 144) = 1.795313$	$L(-8, 8) = 1.871872$
$L(10, 10) = 1.901980$	$L(11, 11) = 1.828405$	$L(11, 10) = 2.116032$
$L(9, 11) = 1.596263$	$L(12, 10) = 1.696240$	$L(9, 12) = 1.713803$
$L(10, 12) = 1.690240$	$L(9, 11) = 1.596263$	$L(10, 11) = 2.116032$
$L(14, 14) = 1.580554$	$L(15, 15) = 1.595900$	$L(15, 14) = 1.876807$
$L(13, 15) = 1.799331$	$L(16, 14) = 1.786943$	$L(13, 15) = 2.15444$
$L(14, 16) = 1.786943$	$L(13, 15) = 1.799331$	$L(14, 15) = 1.876807$

HEXAGON DIAGONALS

SHORT DIAGONALS

L(18, 18) = 1.686868	L(19, 19) = 1.844545	L(19, 18) = 2.116699
L(17, 19) = 1.619581	L(20, 18) = 1.676884	L(17, 20) = 1.717877
L(18, 20) = 1.676884	L(17, 19) = 1.619581	L(18, 19) = 2.116699
L(22, 22) = 1.873219	L(23, 23) = 1.835132	L(23, 22) = 2.108340
L(21, 23) = 1.633622	L(24, 22) = 1.678423	L(21, 24) = 1.743166
L(22, 24) = 1.678423	L(21, 23) = 1.633622	L(22, 23) = 2.108340
L(26, 26) = 1.609640	L(27, 27) = 1.651730	L(27, 26) = 1.912770
L(25, 27) = 1.794617	L(28, 26) = 1.758615	L(25, 28) = 2.157166
L(26, 28) = 1.758615	L(25, 27) = 1.794617	L(26, 27) = 1.912770
L(30, 30) = 1.849318	L(31, 31) = 1.812404	L(31, 30) = 2.086076
L(29, 31) = 1.651463	L(32, 30) = 1.696761	L(29, 32) = 1.803470
L(30, 32) = 1.696761	L(29, 31) = 1.651463	L(30, 31) = 2.086076
L(34, 34) = 1.803664	L(35, 35) = 1.756433	L(35, 34) = 2.041571
L(33, 35) = 1.679684	L(36, 34) = 1.731398	L(33, 36) = 1.910072
L(34, 36) = 1.731398	L(35, 35) = 1.679684	L(34, 35) = 2.041571
L(38, 38) = 1.677888	L(39, 39) = 1.731589	L(39, 38) = 1.976211
L(37, 39) = 1.770055	L(40, 38) = 1.716704	L(37, 40) = 2.044257
L(38, 40) = 1.718704	L(37, 39) = 1.770055	L(38, 39) = 1.976211
L(42, 46) = 1.880021	L(43, 45) = 1.835419	L(43, 46) = 2.189266
L(41, 43) = 1.631654	L(44, 46) = 1.732410	L(41, 44) = 1.736335
L(42, 44) = 1.631654	L(41, 45) = 1.569978	L(42, 45) = 2.026906
L(48, 52) = 1.859420	L(49, 51) = 1.832100	L(49, 52) = 2.215834
L(47, 49) = 1.719695	L(50, 52) = 1.752026	L(47, 50) = 1.765193
L(48, 50) = 1.602919	L(47, 51) = 1.565668	L(48, 51) = 1.975690
L(54, 58) = 1.844260	L(55, 57) = 1.825533	L(55, 58) = 2.224494
L(53, 55) = 1.743198	L(56, 58) = 1.764415	L(53, 56) = 1.789705
L(54, 56) = 1.591857	L(53, 57) = 1.566782	L(54, 57) = 1.945542
L(60, 64) = 1.831893	L(61, 63) = 1.817285	L(61, 64) = 2.224820
L(59, 61) = 1.757549	L(62, 64) = 1.773542	L(59, 62) = 1.812273
L(60, 62) = 1.589543	L(59, 63) = 1.570416	L(60, 63) = 1.925697

LONG DIAGONALS

L(66, 70) = 1.820571 L(67, 69) = 1.807718 L(67, 70) = 2.221246
 L(65, 67) = 1.767942 L(68, 70) = 1.781588 L(65, 68) = 1.834543
 L(66, 68) = 1.591599 L(65, 69) = 1.575073 L(66, 69) = 1.910032
 L(72, 76) = 1.809669 L(73, 75) = 1.797325 L(73, 76) = 2.217184
 L(71, 73) = 1.777236 L(74, 76) = 1.789956 L(71, 74) = 1.856458
 L(72, 74) = 1.594563 L(71, 75) = 1.579007 L(72, 75) = 1.894523
 L(78, 82) = 1.875985 L(79, 81) = 1.841840 L(79, 82) = 2.162714
 L(77, 79) = 1.664843 L(80, 82) = 1.708697 L(77, 80) = 1.735502
 L(78, 80) = 1.642919 L(77, 81) = 1.596318 L(78, 81) = 2.057473
 L(84, 88) = 1.861072 L(85, 87) = 1.832979 L(85, 88) = 2.184486
 L(83, 85) = 1.694258 L(86, 88) = 1.732376 L(83, 86) = 1.764083
 L(84, 86) = 1.625630 L(83, 87) = 1.587891 L(84, 87) = 2.012616
 L(90, 94) = 1.844881 L(91, 93) = 1.819883 L(91, 94) = 2.192878
 L(89, 91) = 1.723479 L(92, 94) = 1.752165 L(89, 92) = 1.797465
 L(90, 92) = 1.619051 L(89, 93) = 1.586399 L(90, 93) = 1.976368
 L(96, 100) = 1.827402 L(97, 99) = 1.803393 L(97, 100) = 2.194472
 L(95, 97) = 1.744705 L(98, 100) = 1.770836 L(95, 98) = 1.833968
 L(96, 98) = 1.618239 L(95, 99) = 1.587767 L(96, 99) = 1.943021
 L(102,106) = 1.808837 L(103,105) = 1.784559 L(103,106) = 2.193904
 L(101,103) = 1.764751 L(104,106) = 1.789731 L(101,104) = 1.871797
 L(102,104) = 1.619115 L(101,105) = 1.589247 L(102,105) = 1.908592
 L(108,112) = 1.860987 L(109,111) = 1.827871 L(109,112) = 2.136855
 L(107,109) = 1.668465 L(110,112) = 1.709590 L(107,110) = 1.771597
 L(108,110) = 1.661145 L(107,111) = 1.618084 L(108,111) = 2.058430
 L(114,118) = 1.842964 L(115,117) = 1.809727 L(115,118) = 2.150671
 L(113,115) = 1.699984 L(116,118) = 1.738546 L(113,116) = 1.812563
 L(114,116) = 1.653691 L(113,117) = 1.611711 L(114,117) = 2.011252
 L(120,124) = 1.820483 L(121,123) = 1.786004 L(121,124) = 2.155965
 L(119,121) = 1.729647 L(122,124) = 1.766876 L(119,122) = 1.861161
 L(120,122) = 1.651823 L(119,123) = 1.609889 L(120,123) = 1.963308
 L(126,130) = 1.828699 L(127,129) = 1.787983 L(127,130) = 2.100247
 L(125,127) = 1.686168 L(128,130) = 1.732523 L(125,128) = 1.852450
 L(126,128) = 1.691664 L(125,129) = 1.643330 L(126,129) = 2.031828
 L(132,136) = 1.800839 L(133,135) = 1.756692 L(133,136) = 2.105737
 L(131,133) = 1.721736 L(134,136) = 1.767888 L(131,134) = 1.911417
 L(132,134) = 1.690583 L(131,135) = 1.640385 L(132,135) = 1.973042

DIHEDRAL ANGLES - EXPRESSED IN DEG

ORDERED TRIPLET	DIHD ANGLE						
145 1 1	179.16245	1 145 1	179.01543	1 10 2	178.05465	1 10 2	178.05465
10 2 1	178.40780	10 1 2	178.36357	2 46 3	177.15195	2 46 3	177.15195
46 3 2	177.71673	46 2 3	177.69527	3 52 4	176.37455	3 52 4	176.37455
52 4 3	177.08613	52 3 4	177.08578	4 58 5	175.71496	4 58 5	175.71496
58 5 4	176.54820	58 4 5	176.55489	5 64 6	175.18702	5 64 6	175.18702
64 6 5	176.10597	64 5 6	176.11788	6 70 7	174.80609	6 70 7	174.80609
70 7 6	175.77699	70 6 7	175.79031	7 76 8	174.57891	7 76 8	174.57891
76 8 7	175.57640	76 7 8	175.58536	8 13 8	174.50389	8 13 8	174.50389
13 8 8	175.51234	12 42 42	177.52220	42 12 42	177.99223	42 12 42	177.99223
44 18 48	176.72148	18 48 44	176.97099	18 44 48	177.48493	18 44 48	177.48493
50 82 54	176.02054	82 54 50	176.46451	82 54 54	176.96049	82 54 54	176.96049
56 88 60	175.43890	88 60 56	176.03829	88 60 56	176.47848	88 60 56	176.47848
62 94 66	174.99593	94 66 62	175.71983	94 62 66	176.07114	94 62 66	176.07114
68 110 72	174.70272	100 72 68	175.52946	100 68 72	175.75876	100 68 72	175.75876
74 116 15	174.55870	106 15 74	175.47602	106 74 74	175.55607	106 74 74	175.55607
20 78 78	176.29120	78 20 78	177.13450	80 22 84	175.70003	80 22 84	175.70003
22 84 80	175.90776	22 80 84	176.72601	86 112 90	175.22936	86 112 90	175.22936
112 90 96	175.61450	112 86 90	176.32329	92 118 96	174.89828	92 118 96	174.89828
118 96 92	175.44140	118 92 96	175.96702	98 124 102	174.70772	98 124 102	174.70772
124 102 98	175.40093	124 98 102	175.68208	104 25 104	174.64650	104 25 104	174.64650
125 104 104	175.48701	24 108 108	175.44969	108 24 108	176.42722	108 24 108	176.42722
110 30 114	175.11539	30 114 110	175.30430	30 110 114	176.11168	30 110 114	176.11168
116 130 120	174.90990	130 120 116	175.28159	130 116 120	175.81513	130 116 120	175.81513
122 136 97	174.81686	136 27 122	175.37540	136 122 27	175.56215	136 122 27	175.56215
32 126 126	175.11393	126 32 126	175.87201	128 34 132	175.02327	128 34 132	175.02327
34 132 128	175.21869	34 128 132	175.62831	134 37 134	174.99869	134 37 134	174.99869
37 134 134	175.40157	36 39 39	175.20687	39 36 39	175.41937	39 36 39	175.41937

Execution time—25 sec

INITIAL GUESS

AG(1)	THRU	AG(152)	=	1.00+001
AG(153)	=	-2.00+001		
AG(154)	=	-2.00+001		
AG(155)	=	-2.00+001		
AG(156)	=	-2.00+001		
AG(157)	=	-2.00+001		
AG(158)	=	-2.00+001		
AG(159)	=	-2.00+001		
AG(160)	=	-2.00+001		
AG(161)	=	-2.00+001		
AG(162)	=	-1.00+001		
AG(163)	=	-1.20+001		

FACE ANGLES -- EXPRESSED AS EXCESS OVER 120 DEG

6 ITERATIONS REQUIRED

INDEX	EXCESS (DEG)	INDEX	EXCESS (DEG)	INDEX	EXCESS (DEG)
1	5.997614+000	56	-1.693297+001	111	1.756303+001
2	8.080611+000	57	1.290407+001	112	-7.152632+000
3	3.452224+000	58	2.934917+000	113	-8.124403+000
4	8.480535+000	59	-1.493604+001	114	1.290630+001
5	8.368684+000	60	1.094693+001	115	-3.006591+000
6	6.171988+000	61	4.798182+000	116	1.218543+001
7	7.944349+000	62	-1.664806+001	117	1.514690+001
8	7.759183+000	63	1.142862+001	118	4.005331+000
9	7.687428+000	64	4.124120+000	119	-9.050714+000
10	2.873199+000	65	-1.480576+001	120	1.081793+001
11	2.397979+001	66	9.799638+000	121	-3.520567-002
12	-4.09615+001	67	5.697603+000	122	-1.287358+001
13	-3.993076+000	68	-1.624423+001	123	1.231657+001
14	1.219866+001	69	1.005602+001	124	-9.930898-001
15	2.133919+001	70	5.142720+000	125	-9.344740+000
16	-1.187041+001	71	-1.452033+001	126	8.152754+000
17	-6.118593+000	72	8.568244+000	127	2.953899+000
18	1.463882+001	73	6.599827+000	128	-1.308539+001
19	-1.482023+001	74	-1.584648+001	129	9.231465+000

Execution time-34 sec

Single precision

FACE ANGLES -- EXPRESSED AS EXCESS OVER 120 DEG

6 ITERATIONS REQUIRED

INDEX	EXCESS (DEG)						
20	8.175120+000	75	8.771915+000	130	2.048338+000	131	-9.373827+000
21	5.367166+000	76	6.184253+000	131	5.147151+000	132	6.059332+000
22	-1.226435+001	77	-1.430484+001	132	-1.311246+001	133	1.358754+001
23	1.928423+001	78	7.334053+000	134	1.359414+000	135	-5.436744+000
24	-1.057992+001	79	7.612128+000	134	-5.694141+000	135	-5.436744+000
25	-5.160038+000	80	-1.559751+001	135	-1.080913+000	136	-8.588602+000
26	1.419570+001	81	1.958545+001	136	-9.964347+000	137	-2.173277+000
27	1.613384+001	82	-7.354727+000	137	1.028769+001	138	-5.673040+000
28	-9.026954+000	83	-9.681899+000	138	-5.225560+000	139	-1.080913+000
29	-4.665239+000	84	1.436452+001	139	2.682000+000	140	-1.507416+000
30	1.125054+001	85	-2.764724+000	140	-1.034893+001	141	-1.34893+001
31	-1.044496+001	86	-1.414862+001	141	-5.673040+000	142	-5.673040+000
32	-4.549029+000	87	1.732525+001	142	-5.673040+000	143	-2.722467+000
33	1.526014+000	88	-3.876208+000	143	-1.99523+001	144	-1.507416+000
34	-5.705128+000	89	-1.150244+001	144	-1.616122+001	145	-1.361546+000
35	2.073738+001	90	1.326841+001	145	-1.690445+001	146	-2.781526+000
36	-6.561010+000	91	-2.158218+001	146	-6.988022+000	147	-6.901210+000
37	-1.455610+000	92	-1.499918+001	147	-1.507416+000	148	-1.199523+001
38	5.295863+000	93	1.504529+001	148	-1.616122+001	149	-1.634398+001
39	2.012821+001	94	-1.195445+000	149	-1.690445+001	150	-1.588870+001
40	-6.115614+000	95	-1.225845+001	150	-1.551837+001	151	-1.696107+001
41	-1.069017+001	96	-1.168348+001	151	-1.673737+001	152	-1.537486+001
42	1.322957+001	97	1.915782+000	152	-1.59465+001	153	-1.873199+000
43	4.765690+002	98	-1.519065+001	153	-1.616122+001	154	-1.200000+001
44	-1.656966+001	99	1.276402+001	154	-1.690445+001	155	-1.690445+001
45	1.683987+001	100	1.059517+001	155	-1.690445+001	156	-1.696107+001
46	-1.411108+000	101	-1.241442+001	156	-1.673737+001	157	-1.634398+001
47	-1.352822+001	102	9.751087+000	157	-1.588870+001	158	-1.588870+001
48	1.280753+001	103	3.919185+000	158	-1.588870+001	159	-1.588870+001
49	2.284141+000	104	-1.507938+001	159	-1.588870+001	160	-1.588870+001
50	-4.699221+001	105	1.046109+001	160	-1.588870+001	161	-1.537486+001
51	1.460630+001	106	3.190540+000	161	-1.232785+001	162	-2.873199+000
52	1.266563+000	107	-1.232785+001	162	-1.464465+001	163	-1.200000+001
53	-1.464465+001	108	7.594654+000	163	-1.490075+001	164	-1.490075+001
54	1.196632+001	109	5.982316+000	164	-1.738428+000	165	-1.738428+000
55	5.738428+000	110	-1.490075+001	165	-1.490075+001		

HEXAGON DIAGONALS

SHORT DIAGONALS

$L(-1, -1) = 1.618083$	$L(-1, -1) = 1.618083$	$L(-1, -1) = 1.902155$
$L(153, -1) = 1.781994$	$L(153, -1) = 1.781994$	$L(153, 153) = 2.175503$
$L(-1, 153) = 1.781994$	$L(-1, 153) = 1.781994$	$L(-1, 1) = 1.902155$
$L(-2, -2) = 1.574288$	$L(-2, -2) = 1.574288$	$L(-2, -2) = 1.865042$
$L(154, -2) = 1.798204$	$L(154, -2) = 1.798204$	$L(154, 154) = 2.233539$
$L(-2, 154) = 1.798204$	$L(-2, 154) = 1.798204$	$L(-2, 2) = 1.865042$
$L(-3, -3) = 1.566254$	$L(-3, -3) = 1.566254$	$L(-3, -3) = 1.858266$
$L(155, -3) = 1.801034$	$L(155, -3) = 1.801034$	$L(155, 155) = 2.243724$
$L(-3, 155) = 1.801034$	$L(-3, 155) = 1.801034$	$L(-3, 3) = 1.858266$
$L(-4, -4) = 1.565639$	$L(-4, -4) = 1.565639$	$L(-4, -4) = 1.857748$
$L(156, -4) = 1.801249$	$L(156, -4) = 1.801249$	$L(156, 156) = 2.244497$
$L(-4, 156) = 1.801249$	$L(-4, 156) = 1.801249$	$L(-4, 4) = 1.857748$
$L(-5, -5) = 1.568066$	$L(-5, -5) = 1.568066$	$L(-5, -5) = 1.859793$
$L(157, -5) = 1.800400$	$L(157, -5) = 1.800400$	$L(157, 157) = 2.241439$
$L(-5, 157) = 1.800400$	$L(-5, 157) = 1.800400$	$L(-5, 5) = 1.859793$
$L(-6, -6) = 1.572318$	$L(-6, -6) = 1.572318$	$L(-6, -6) = 1.863380$
$L(158, -6) = 1.798902$	$L(158, -6) = 1.798902$	$L(158, 158) = 2.236048$
$L(-6, 158) = 1.798902$	$L(-6, 158) = 1.798902$	$L(-6, 6) = 1.863380$
$L(-7, -7) = 1.577217$	$L(-7, -7) = 1.577217$	$L(-7, -7) = 1.867515$
$L(159, -7) = 1.79162$	$L(159, -7) = 1.79162$	$L(159, 159) = 2.229792$
$L(-7, 159) = 1.79162$	$L(-7, 159) = 1.79162$	$L(-7, 7) = 1.867515$
$L(-8, -8) = 1.581183$	$L(-8, -8) = 1.581183$	$L(-8, -8) = 1.870866$
$L(160, -8) = 1.795742$	$L(160, -8) = 1.795742$	$L(160, 160) = 2.224688$
$L(-8, 160) = 1.795742$	$L(-8, 160) = 1.795742$	$L(-8, 8) = 1.870866$
$L(-9, -9) = 1.582715$	$L(-9, -9) = 1.582715$	$L(-9, -9) = 1.872161$
$L(161, -9) = 1.795190$	$L(161, -9) = 1.795190$	$L(161, 161) = 2.222707$
$L(-9, 161) = 1.795190$	$L(-9, 161) = 1.795190$	$L(-9, 9) = 1.872161$
$L(-10, -10) = 1.706436$	$L(-10, -10) = 1.706436$	$L(-10, 10) = 1.999371$
$L(-10, 10) = 1.706436$	$L(-10, 10) = 1.706436$	$L(-10, 162) = 1.999371$
$L(-162, 162) = 1.756577$	$L(-162, 162) = 1.756577$	$L(-162, 10) = 1.999371$

HEXAGON DIAGONALS

SHORT DIAGONALS		LONG DIAGONALS	
L(12, 12) = 1.902004	L(13, 13) = 1.828498	L(13, 12) = 2.116084	
L(11, 13) = 1.596261	L(14, 12) = 1.696160	L(11, 14) = 1.713661	
L(12, 14) = 1.696160	L(11, 13) = 1.596261	L(12, 13) = 2.116084	
L(16, 16) = 1.887251	L(17, 17) = 1.845338	L(17, 16) = 2.117219	
L(15, 17) = 1.619362	L(18, 16) = 1.676213	L(15, 18) = 1.716389	
L(16, 18) = 1.676213	L(15, 17) = 1.619362	L(16, 17) = 2.117219	
L(20, 20) = 1.588615	L(21, 21) = 1.615318	L(21, 20) = 1.888417	
L(19, 21) = 1.798926	L(22, 20) = 1.776972	L(19, 22) = 2.197077	
L(20, 22) = 1.776972	L(19, 21) = 1.798926	L(20, 21) = 1.888417	
L(24, 24) = 1.675076	L(25, 25) = 1.842342	L(25, 24) = 2.110576	
L(23, 25) = 1.632478	L(26, 24) = 1.675818	L(23, 26) = 1.736908	
L(24, 26) = 1.675818	L(23, 25) = 1.632478	L(24, 25) = 2.110576	
L(28, 28) = 1.855242	L(29, 29) = 1.821732	L(29, 28) = 2.092786	
L(27, 29) = 1.647986	L(30, 28) = 1.689915	L(27, 30) = 1.786086	
L(28, 30) = 1.689915	L(27, 29) = 1.647986	L(28, 29) = 2.092786	
L(32, 32) = 1.633837	L(33, 33) = 1.680139	L(33, 32) = 1.935219	
L(31, 33) = 1.786343	L(34, 32) = 1.745214	L(31, 34) = 2.118964	
L(32, 34) = 1.745214	L(31, 33) = 1.786343	L(32, 33) = 1.935219	
L(36, 36) = 1.818017	L(37, 37) = 1.776400	L(37, 36) = 2.056581	
L(35, 37) = 1.671988	L(38, 36) = 1.719209	L(35, 38) = 1.876018	
L(36, 38) = 1.719209	L(35, 37) = 1.671988	L(36, 37) = 2.056581	
L(40, 44) = 1.880149	L(41, 43) = 1.835714	L(41, 44) = 2.189496	
L(39, 41) = 1.676241	L(42, 44) = 1.732205	L(39, 42) = 1.735817	
L(40, 42) = 1.631365	L(39, 43) = 1.569881	L(40, 43) = 2.027048	
L(46, 50) = 1.859809	L(47, 49) = 1.832778	L(47, 50) = 2.216476	
L(45, 47) = 1.719606	L(48, 50) = 1.751638	L(45, 48) = 1.763920	
L(46, 48) = 1.602213	L(45, 49) = 1.565301	L(46, 49) = 1.975969	
L(52, 56) = 1.845119	L(53, 55) = 1.826852	L(53, 56) = 2.225838	
L(51, 53) = 1.742998	L(54, 56) = 1.763747	L(51, 54) = 1.787185	
L(52, 54) = 1.590475	L(51, 55) = 1.565944	L(52, 55) = 1.946061	

$L(58, 62) = 1.833452 \quad L(59, 61) = 1.819538 \quad L(59, 62) = 2.227021$
 $L(57, 59) = 1.757092 \quad L(60, 62) = 1.772392 \quad L(57, 60) = 1.808003$
 $L(58, 60) = 1.587386 \quad L(57, 61) = 1.569033 \quad L(58, 61) = 1.926770$

 $L(64, 68) = 1.823012 \quad L(65, 67) = 1.811135 \quad L(65, 68) = 2.224044$
 $L(63, 65) = 1.766911 \quad L(66, 68) = 1.779611 \quad L(63, 66) = 1.828237$
 $L(64, 66) = 1.588768 \quad L(63, 67) = 1.573394 \quad L(64, 67) = 1.912346$

 $L(-70, -74) = 1.813029 \quad L(-71, -73) = 1.801914 \quad L(-71, -74) = 2.219768$
 $L(69, 71) = 1.75170 \quad L(72, 74) = 1.786741 \quad L(69, 72) = 1.848287$
 $L(-70, 72) = 1.591789 \quad L(-69, 73) = 1.577670 \quad L(-70, 73) = 1.899071$

 $L(-76, -80) = 1.803453 \quad L(-77, -79) = 1.792463 \quad L(-77, -80) = 2.216776$
 $L(75, 77) = 1.783471 \quad L(78, 80) = 1.794610 \quad L(75, 78) = 1.867396$
 $L(-76, 78) = 1.594063 \quad L(-75, 79) = 1.580337 \quad L(-76, 79) = 1.884448$

 $L(-82, -86) = 1.876898 \quad L(-83, -85) = 1.843486 \quad L(-83, -86) = 2.163892$
 $L(81, 83) = 1.664347 \quad L(84, 86) = 1.707422 \quad L(81, 84) = 1.732320$
 $L(-82, -84) = 1.641482 \quad L(-81, -85) = 1.595709 \quad L(-82, -85) = 2.058553$

 $L(-88, -92) = 1.862908 \quad L(-89, -91) = 1.835983 \quad L(-89, -92) = 2.186622$
 $L(-87, -89) = 1.697240 \quad L(-90, -92) = 1.730164 \quad L(-87, -90) = 1.758301$
 $L(-88, -90) = 1.623123 \quad L(-87, -91) = 1.586715 \quad L(-88, -91) = 2.014749$

 $L(-94, -98) = 1.848061 \quad L(-95, -97) = 1.824837 \quad L(-95, -98) = 2.196057$
 $L(93, 95) = 1.721525 \quad L(96, 98) = 1.748526 \quad L(93, 96) = 1.788170$
 $L(-94, -96) = 1.615378 \quad L(-93, -97) = 1.584679 \quad L(-94, -97) = 1.980441$

 $L(100, 104) = 1.832474 \quad L(101, 103) = 1.810775 \quad L(101, 104) = 2.198323$
 $L(99, 101) = 1.741223 \quad L(102, 104) = 1.765233 \quad L(99, 102) = 1.820625$
 $L(-100, 102) = 1.613772 \quad L(-99, 103) = 1.585863 \quad L(100, 103) = 1.950324$

 $L(106, 110) = 1.816002 \quad L(107, 109) = 1.794476 \quad L(107, 110) = 2.197671$
 $L(105, 107) = 1.759219 \quad L(108, 110) = 1.781873 \quad L(105, 108) = 1.854623$
 $L(106, 108) = 1.614664 \quad L(105, 109) = 1.587761 \quad L(106, 109) = 1.920357$

 $L(112, 116) = 1.864414 \quad L(113, 115) = 1.833486 \quad L(113, 116) = 2.140569$
 $L(111, 113) = 1.666300 \quad L(114, 116) = 1.705220 \quad L(111, 114) = 1.760882$
 $L(112, 114) = 1.656860 \quad L(111, 115) = 1.616130 \quad L(112, 115) = 2.062678$

HEXAGON DIAGONALS

SHORT DIAGONALS

$L(118,122) = 1.848739 \quad L(119,121) = 1.818602 \quad L(119,122) = 2.155910$
 $L(117,119) = 1.696047 \quad L(120,122) = 1.731743 \quad L(117,120) = 1.796171$
 $L(118,120) = 1.647751 \quad L(117,121) = 1.609025 \quad L(118,121) = 2.018982$

$L(124,128) = 1.629531 \quad L(125,127) = 1.798755 \quad L(125,128) = 2.162368$
 $L(123,125) = 1.723320 \quad L(126,128) = 1.757250 \quad L(123,126) = 1.838677$
 $L(124,126) = 1.644837 \quad L(123,127) = 1.606827 \quad L(124,127) = 1.976156$

$L(130,134) = 1.806906 \quad L(131,133) = 1.775206 \quad L(131,134) = 2.164321$
 $L(129,131) = 1.749648 \quad L(132,134) = 1.782483 \quad L(129,132) = 1.884973$
 $L(130,132) = 1.644548 \quad L(129,133) = 1.606545 \quad L(130,133) = 1.931569$

$L(136,140) = 1.838185 \quad L(137,139) = 1.802068 \quad L(137,140) = 2.109378$
 $L(135,137) = 1.680243 \quad L(138,140) = 1.722541 \quad L(135,138) = 1.827306$
 $L(136,138) = 1.682675 \quad L(135,139) = 1.638661 \quad L(136,139) = 2.043426$

$L(142,146) = 1.814732 \quad L(143,145) = 1.775836 \quad L(143,146) = 2.116918$
 $L(141,143) = 1.712775 \quad L(144,146) = 1.754979 \quad L(141,144) = 1.878934$
 $L(142,144) = 1.680442 \quad L(141,145) = 1.634803 \quad L(142,145) = 1.990975$

$L(148,152) = 1.789775 \quad L(149,151) = 1.743810 \quad L(149,152) = 2.061916$

$L(147,149) = 1.707806 \quad L(150,152) = 1.755812 \quad L(147,150) = 1.935371$

$L(148,150) = 1.718747 \quad L(147,151) = 1.668723 \quad L(148,151) = 1.997638$

LONG DIAGONALS

DIHEDRAL ANGLES -- EXPRESSED IN DEG

			DIHEDRAL ANGLE								
			ORDERED TRIPLET								
-163	1	1	179.24179	1	163	1	179.10869	1	12	2	178.23868
12	2	1	178.55848	12	1	2	178.51837	2	44	3	177.41957
44	3	2	177.92263	44	2	3	177.91197	3	50	4	176.70920
50	4	3	177.35657	50	3	4	177.35553	4	56	5	176.09689
56	5	4	176.85935	56	4	5	176.86421	5	62	6	175.59193
62	6	5	176.43951	62	5	6	176.44915	6	68	7	175.20754
68	7	6	176.11057	68	6	7	176.12267	7	74	8	174.95176
74	8	7	175.88575	74	7	8	175.89609	8	80	9	174.82515
80	9	8	175.77388	80	8	9	175.77798	14	40	40	177.75558
40	14	40	178.18173	42	16	46	177.02620	16	46	42	177.25256
16	42	46	177.72048	48	86	52	176.38024	86	52	48	176.78411
86	48	52	177.24019	54	92	58	175.83161	92	58	54	176.37887
92	54	58	176.79245	60	98	64	175.39602	98	64	60	176.05889
98	60	64	176.40456	66	104	70	175.08493	104	70	66	175.84206
104	66	70	176.09382	72	110	76	174.90105	110	76	72	175.73906
110	72	76	175.87263	78	19	78	174.84064	19	78	78	175.75127
118	82	82	176.62717	82	18	82	177.40019	84	24	88	176.07228
24	88	84	176.26106	24	84	88	177.02365	90	116	94	175.61426
116	94	90	175.96397	116	90	94	176.64705	96	122	100	175.27156
122	100	96	175.76247	122	96	100	176.30644	102	128	106	175.04992
128	106	102	175.67146	128	102	106	176.02390	108	134	21	174.94329
134	21	108	175.69216	134	108	21	175.81448	26	112	112	175.81519
112	26	112	176.74535	144	28	118	175.46997	28	118	114	175.63969
28	114	118	176.44788	120	140	124	175.23615	140	124	120	175.56601
140	120	124	176.16407	126	146	130	175.10710	146	130	126	175.59731
146	126	130	175.91594	132	131	132	175.06688	31	132	132	175.72071
30	136	136	175.41816	136	30	136	176.22147	138	36	142	175.29276
36	142	138	175.46306	36	138	142	175.99094	144	152	33	175.24064
152	33	144	175.59023	152	144	33	175.77431	38	148	148	175.42542
148	38	148	175.79988	150	162	150	175.41823	162	150	150	175.60445

Execution time - 34 sec



Security Classification

DOCUMENT CONTROL DATA - R & D

(Security classification of title, body of abstract and indexing annotation must be entered when the overall report is classified)

1. ORIGINATING ACTIVITY (Corporate author) Naval Research Laboratory Washington, D.C. 20390	2a. REPORT SECURITY CLASSIFICATION Unclassified
2b. GROUP	

3. REPORT TITLE

A PARTICULAR CLASS OF PENTI-HEXAGONAL POLYHEDRA

4. DESCRIPTIVE NOTES (Type of report and inclusive dates)

A final report on one phase of the problem; work on other phases continues.

5. AUTHOR(S) (First name, middle initial, last name)

Gerald Chayt and Herbert Hauptman

6. REPORT DATE August 22, 1968	7a. TOTAL NO. OF PAGES 106	7b. NO. OF REFS None
8a. CONTRACT OR GRANT NO. NRL Problem B01-03	9a. ORIGINATOR'S REPORT NUMBER(S) NRL Report 6706	
b. PROJECT NO. RR 003-05-41-5060		
c.		
d.		

10. DISTRIBUTION STATEMENT

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11. SUPPLEMENTARY NOTES	12. SPONSORING MILITARY ACTIVITY Department of the Navy (Office of Naval Research), Washington, D.C. 20360
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13. ABSTRACT

In accordance with the Navy's interest in developing an underwater spherical vessel capable of human occupancy, studies have been made of a particular class of convex polyhedra as a possible approximation to a sphere. The class referred to is a category of penti-hexagonal polyhedra, formed from the regular dodecahedron by inserting successive layers of equilateral convex hexagons within the basic pentagonal structure in a manner which maximizes the congruence and the symmetry of the polyhedral surfaces.

The structure of a particular polyhedron in this class is determined by computing the vertex angles of all the distinct hexagons composing its surface, i.e., the polyhedron face angles. With this end in mind, trigonometric equations are derived for all hexagons having less than a certain degree of symmetry, and all dihedral angles whose edges are not perpendicular to a plane of symmetry of the polyhedron. These equations are then solved using a Newton-Raphson-type process. The structure of all polyhedra in this class up to 3242 faces has been determined, and the number of trigonometric equations in each case exactly equals the number of unknown angles. It is suspected that this equality is maintained for higher order polyhedra in this class.

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Security Classification

14. KEY WORDS	LINK A		LINK B		LINK C	
	ROLE	WT	ROLE	WT	ROLE	WT
Mathematics Convex polyhedra Penti-hexagonal polyhedra Newton-Raphson approximation Polyhedral structures Digital computer programs						

After the relevant trigonometric equations and the techniques for solving them have been derived, it is seen that, in practically all instances, the value of the solution, if convergence occurs, is independent of the initial guess used for the face angles.

Trends among certain face angles become evident for progressively higher order PH polyhedra, which permit calculating approximate diameters as a multiple of the edge length; for polyhedra having up to 3242 faces, the values of all face angles, hexagon diagonals, and dihedral angles may also be determined.

Two conjectures emerge from the study, and these take the form of two theorems which are based on six preliminary definitions.